Partnering With a Savvy Agent*

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Abstract

This paper analyzes the problem of a principal contracting with an agent to form a short-lived partnership to exploit an asset before reselling. The agent is *savvy* in the sense that he privately observes the resale value of the asset before negotiating with the principal to dissolve the partnership. The principal is not savvy but can "neutralize" the agent's informational advantage and have him disclose the resale value *for free* by dissolving the partnership by means of a Texas shootout with the agent in the role of the proposer. Thus, in the optimal contract, the agent's savvy does not distort the allocation of shares. A higher ex-ante aggregate surplus can be attained if the principal commits to giving the asset away to the agent upon dissolution: She earns a lower revenue but lets all types of the agent enjoy a higher surplus. However, the additional surplus for lower types is insufficient to compensate the principal, so this higher ex-ante aggregate surplus is unattainable at the interim stage.

Keywords: Partnerships, dynamic mechanism design, endogenous information structure

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1 Introduction

A non-tech-savvy investor has an idea for a mobile application. To develop said application, the investor must partner with a programmer who possesses the right expertise. If the tech-savvy partner exerts himself, he may be able to bring said application to fruition and raise the value of the venture, both present and future. On the other hand, his savvy grants him a better understanding of tech market trends, which can translate into a more-accurate appraisal of the future value of the venture. This informational asymmetry can leave the investor at a disadvantage in future negotiations with her partner. For instance, if the partnership is to be dissolved, the programmer has a better estimate of the value of the shares, which may enable him to capture a larger cut of the surplus at the investor's expense. What is the best structure and terms of the partnership for the investor to incentivize the programmer both to work hard in developing the application and to share his expertise?

Another investor acquires a farm. Being an outsider to farming, the investor must partner with a farmer to exploit it. The farmer has the required know-how to operate the farm equipment but he also has a better grasp of the production cycle and the trends in the market for produce than the investor. What should be the terms of the partnership contract to both motivate the farmer to exploit the land and to elicit their expert assessment of future produce prices?

These examples share the following two key components: (i) The principal has a business opportunity or owns an asset that she can neither accurately appraise nor exploit on her own; (ii) the agent, as a partner to the principal, has savvy that enables him to both exploit the asset and to obtain a more-precise appraisal of its future value. The latter aspect of his savvy may give the agent an advantage over the principal in future negotiations, in particular in dissolving the partnership.

This paper analyzes the principal's contracting problem with a savvy agent. The principal must decide how many shares to sell to the agent in order to motivate him to exert effort and to share his savvy, accounting for his informational advantage in future negotiations. To simplify the latter analysis, we assume that the partnership lasts for only one exploitation period and must then be dissolved; thus, we identify all future negotiations between the partners with the negotiations to dissolve the partnership.

The agent's savvy consists of two pieces of valuable information: (a) How much profit he can raise within the partnership, which we call the *exploitation value*; and (b) the *dissolution value*, which we identify with the asset's resale value and represents the amount over which the partners bargain in the dissolution negotiation. The two values are correlated, their joint distribution being common knowledge; yet only the agent observes the realization of the dissolution value before the dissolution negotiation.

Despite the agent's more-accurate assessment of the dissolution value, the non-savvy principal can "neutralize" this informational advantage and incentivize full disclosure of this piece of information *for free* by having the partnership dissolved through a Texas shootout (Brooks et al., 2010) in which the agent takes on the role of proposer. With the agent proposing a price per share, the principal randomizes between selling her shares and buying her partner's shares in a way that renders the latter's expected payoff constant; thus, the agent "might as well" call a price equal to the actual privately-observed dissolution value. As a result, in the optimal contract, the informational asymmetry regarding this piece of information does not distort the allocation of shares; the only distortion due to information rents for the agent is from the exploitation value signal.

If the agent possesses some bargaining power, he may object to the clause in the contract that calls for him to be the proposer in the dissolution negotiation. However, with the principal as the proposer, a type of *ratchet-effect* phenomenon arises. If the principal believes that the agent is truthful in reporting the exploitation value, she will be tempted to update her assessment of the dissolution value and adjust the terms of the dissolution of the partnership based on the report; but if the agent anticipates that the principal will take his report at face value and use it to set the future price per share, he will be tempted to misrepresent the exploitation value in the first place. Thus, incentive compatibility is undermined.

To avoid this issue, we consider the benchmark where the principal can commit ex-ante to a fixed dissolution price per share. The partners can attain a higher ex-ante aggregate surplus if the principal commits to a price equal to 0, essentially giving the asset away to the agent upon dissolving the partnership: The principal losses revenue but lets all types of the agent enjoy a sufficiently higher surplus. Intuitively, making the agent full owner maximizes how much effort he exerts during the exploitation stage. The principal could capture this surplus back from the agent; however, at the interim stage, the gain in surplus for lower types of the agent is insufficient to compensate the principal for her loss in expected revenue.

The economics literature on partnerships is vast, but it is largely focused on the problem of dissolving existing partnerships. The seminal paper is Cramton et al.

(1987), which identifies the class of initial ownership structures that allow for an efficient dissolution of partnerships in an IPV environment. This analysis has been extended in many directions. Moldovanu (2002) provides a survey of the literature on efficient partnership dissolution under interdependent values. Jehiel and Pauzner (2006) analyzes the case where only one of the partners is informed. Kittsteiner (2003) challenges the efficiency of double auctions for dissolving partnerships in an IPV setting when participation is voluntary, and Ornelas and Turner (2007) introduces control as a variable in the analysis of efficient dissolutions of partnerships.

Loertscher and Wasser (2019) analyzes the more-general problem of how to *best* dissolve *any* given partnership, not only from the point of view of efficiency but also from the point of view of revenues. Solving this more-general problem allows them to find the optimal *ex-ante* ownership structure, by maximizing the expected surplus as a function of the vector of initial shares. Unlike Loertscher and Wasser (2019), we analyze the partnership-constitution problem at the *interim* stage.

Most of the literature on partnership constitution stems from the network and coalitional-game literatures. Bloch et al. (2019) analyzes the problem of forming partnerships to exchange favors in a network. Talman and Yang (2011) formulates the problem of forming partnerships as an assignment problem, proposes an equilibrium notion based on stability, and analyzes the problem of existence of such equilibria. Gudmundsson (2011) introduces the stable-partnership problem, a variation of the model of Talman and Yang (2011), while Gudmundsson (2013) characterizes the core of the partnership-formation problem of Talman and Yang (2011). Finally, Andersson et al. (2013) proposes an algorithm that either finds a unique equilibrium or proves that no equilibria exist.

The agent's savvy in our paper is analogous to the neighbor bidder's insight in oil drilling auctions (Wilson, 1969). In Francetich (2015), this savvy is not "innate," but acquired endogenously. A principal holds two sequential auctions, for two oil tracts, with two agents. In each auction, drilling rights can be awarded in full to either agent or split in half and "double sourced." Whomever wins the auction in the first round gets to privately observe the likelihood of finding oil in the neighboring tract before its auction. In the present paper, our principal faces a single agent who is innately savvy. The structure of the partnership does not affect the agent's savvy but may influence the stakes and bargaining power in future negotiations.

Che et al. (2017) analyzes the problem of a principal procuring an innovative project. The value of the project depends on the innovator's hidden effort. Once the project is procured, the principal holds an auction where firms with hidden costs—

including the innovator themselves—compete for the right to implement the project. While closely related, there are (at least) two key differences with the present paper. Rather than picking an agent from a pool to procure and implement a project, our principal faces only one agent and decides how to split ownership with him. More importantly, both of our value signals are private to the agent, as in Francetich (2015), while the project-value signal in Che et al. (2017) is public.

Cao (2018) is the closest paper to ours, analyzing the problem of constitution and renegotiation of a fixed-horizon dynamic partnership.[1] Every period, partners participate in a mechanism that determines the allocation of shares and the effort recommendations for said period. The paper characterizes a constrained-efficient mechanism to divide profit—both production and dissolution profit. The central difference between Cao (2018) and the present paper is that our mechanism designer is one of the partners themselves, the principal, whose objective is to maximize her own payoff.[2] Thus, our partners are asymmetric both informationally and in terms of bargaining power: Only the agent has private information and exerts effort, while the principal determines the structure and terms of the partnership.

The rest of the paper is organized as follows. Section 2 describes the formal setup. Section 3 analyzes the problem of dissolving an existing partnership (3.1), exploiting the asset within a given partnership (3.2), and constituting the partnership (3.3 and, when the principal has a positive outside option, 3.4). In Section 4, the principal can commit ex-ante to a fixed dissolution price. Section 5 exemplifies generalizations of the structure on the cost of effort (5.1) and correlation (5.2), and discusses the issues of private dissolution values (5.3) and multi-period partnerships (5.4). Section 6 concludes. Proofs and additional details on Example 4 are in the appendix.

2 The Model

A principal ("she") has a business opportunity or owns an asset that she can only exploit by partnering with an agent ("he"). The value the agent can bring to the partnership, if successful, is given by the *exploitation value* $v \in [0, \overline{v}] \subseteq \mathbb{R}_+$, which is drawn from a random variable V with density $f(v)$. We can think of v as the scale of the agent's potential contribution to the partnership. His outside option is 0.

[1] Cetemen (2018) also analyzes a fixed-horizon dynamic partnership. However, in Cetemen (2018), shares are fixed and the analysis focuses on equilibrium effort.

[2] Ours is a common-value partnership, so efficiency is a moot issue.

In order to be successful, the agent must exert costly effort. If effort $e \in [0,1]$ is exerted and the exploitation-value signal is v, the expected value created in the course of the partnership is ev.[3] If v is the highest *potential* output value, then e determines the fraction of this potential value that is realized.

The partnership is *short lived* in the sense that it is to be dissolved after one period of exploitation; upon dissolution, the asset is to be resold.[4] The *dissolution value* is denoted by $w \in \mathbb{R}_+$ and it is positively correlated with the realized exploitation value ev, so v is "good news" for w provided that the agent exerts some effort. In particular, we assume that w is drawn from a random variable W with conditional expectation given ev, $\phi(ev) = E(W|ev)$, having a linear structure:

$$\phi(ev) = \alpha ev; \tag{1}$$

$\alpha > 0$ is the increase in expected dissolution value per additional dollar in realized exploitation value—in other words, how much the agent's involvement appreciates the asset. This specification obtains, for instance, when the dissolution value equals the realized exploitation value plus a 0-mean noise representing uncertain market conditions.[5]

These assumptions on the signal structure of the problem are summarized below.

Assumption 1 (Signal structure)**.** (a) *The distribution of v has full-support density $f(v)$ on $[0,\overline{v}]$.* (b) *The conditional expectation of w given ev is given by* (1) *for some $\alpha > 0$.*

While all the distributions and parameters are common knowledge, the agent is *savvy* in the sense that he privately observes the realization of w before negotiating the dissolution of the partnership with the principal. For instance, the programmer may be able to "extract" more information about the future value of the application than the investor from the same report or piece of news from the tech industry.

[3] More generally, we can take $e \in [\underline{e}, \overline{e}]$ and a strictly increasing function $\mu : [\underline{e}, \overline{e}] \to [0,1]$ such that the expected exploitation value is $\mu(e)v$.

[4] Making the partnership last only one period serves mainly for analytical simplicity, to identify all future negotiations between the partners with the dissolution negotiation and to highlight the role of asymmetric information on the terms of the constitution of the partnership. A similar analysis applies to partnerships that are to last a definite number of multiple periods, or when there is an exogenous probability of dissolution. The substance behind this assumption is that the dissolution is triggered exogenously. If the decision whether to dissolve the partnership now or wait is made conditional on the agent's report of the current resale value, we have a stopping problem embedded in the partnership contracting problem; see the discussion in Section 5.4.

[5] Alternative distributional assumptions that lead to structure (1) are provided in the examples below. Section 3.4 analyzes the case where $\phi(0) > 0$, so the principal has a positive outside option.

Assumption 2 (Agent's savvy)**.** *The agent privately observes both v, before the partnership is constituted, and w, before the partnership is to be dissolved.*[6]

Exploitation of the asset entails the agent exerting costly effort. The cost of effort depends on v as a proxy for the scale of the venture; it is given by a function $c(e, v)$ with the following properties: (a) $c(0, v) = 0$ for all $v \in [0, \overline{v}]$; (b) $c(e, v)$ is strictly increasing in both e and v, and strictly convex in e; and (c) $c(e, v)$ is differentiable and has non-negative cross partial derivative. Throughout most of the paper, we will focus on the following functional form:[7]

$$c(e, v) = (v + \delta\phi(v))\,\frac{e^2}{2} = \frac{1+\delta\alpha}{2} v e^2, \tag{2}$$

where $\delta \in (0, 1)$ is the common discount factor. Effort is non-contractible; the lack of savvy prevents the principal from adequately verifying the agent's effort.

Figure 1 describes the timing of the problem. In the *constitution stage*, the principal allocates shares to the agent in exchange for a payment. In the *exploitation stage*, the agent exerts effort by which the exploitation value is realized. Finally, in the *dissolution stage*, the partners negotiate to dissolve the partnership. We assume that the constitution and production stages are consecutive within the same period, so only dissolution payoffs are discounted.

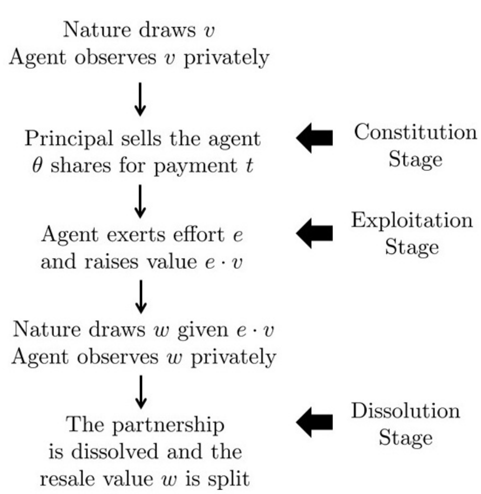


Figure 1: Timing of the partnering problem

[6]Alternatively, we could model the agent's savvy by having him observe a private signal s about the state of the market, so that his assessment of the resale value is $E(W|ev, s)$. The substance behind Assumption 2 is that the agent is better informed about w than is the principal herself.

[7]Section 5.1 exemplifies alternative cost functions.

3 Constituting and Dissolving a Partnership

This section analyzes the principal's problem of constituting a partnership with the agent, who is then to exert effort to exploit the asset before the partnership is to be dissolved. We begin by tackling the problem of dissolving the partnership and work our way backwards to the design of the constitution contract.

3.1 Dissolving an existing partnership

If both partners were equally savvy, they would both assess the dissolution value to be w; there would be no need to employ any information about e or v to assess the asset. In this benchmark, if the agent has been awarded θ shares, he enjoys a dissolution surplus of θw while the principal captures $(1-\theta)w$ for herself.

In spite of the agent's savvy, the principal can attain her benchmark dissolution payoff by having the partnership dissolved through a *Texas-shootout* mechanism, also known as *cake-cutting* or *buy-sell* mechanism, with the agent in the role of the offeror. In a Texas Shootout, a designated proposer calls a price p per share while their partner decides whether to buy or sell at said price.[8] With the agent designated as the proposer, in equilibrium, the principal randomizes between selling her own shares and buying the agent's shares with probabilities θ and $1-\theta$, respectively. This randomization renders the agent's dissolution payoff constant in the choice of p, so he has nothing to lose by calling price $p = w$; see Proposition 1 in Brooks et al. (2010). Thus, the principal can extract w from the agent *for free*. Equilibrium payoffs are θw for the agent and $(1-\theta)w$ for the principal, as in the benchmark.

We summarize this analysis in the next proposition.

Proposition 1 (Optimal dissolution)**.** *The optimal mechanism for the principal to dissolve a partnership where the agent has θ shares is a Texas shootout with the agent as proposer. If the resale value is w, the agent proposes the price $p^* = w$; the principal randomizes between selling and buying with probabilities θ and $1-\theta$, respectively.*

3.2 Production within a partnership

At the production stage, a type-v agent who holds θ shares and exerts effort e assesses his dissolution payoff to be $\theta\alpha e v$. Recall that effort is non-contractible, and

[8]The Texas Shootout mechanism in IPV environments is analyzed in McAfee (1992), and more recently in De-Frutos and Kittsteiner (2008) and Khoroshilov (2017). For this mechanism in common-value environments, as in the present paper, see Brooks et al. (2010) and Li et al. (2013).

so the amount of shares allocated to him, θ, and the payment t made to the principal in the constitution stage, are both independent of effort. For a general cost function $c(e, v)$, production payoff is thus:

$$\mathrm{u}_A(e, \theta, t, v) := \theta ev + \delta\theta\alpha ev - t - c(e, v) = \theta v(1 + \delta\alpha)e - t - c(e, v).$$

The agent's optimal choice of effort, $e^*(\theta, v)$, is $e^*(\theta, v) = (c'_e(\cdot, v))^{-1}((1 + \delta\alpha)\theta v)$. Allocating more shares to the agent allows him to capture more of the value he helps realize, which motivates him to exert higher effort. The effect of v on the choice of e, however, is ambiguous. On the one hand, a higher v means a higher potential value for the partnership, which motivates higher effort. On the other hand, a larger-scale venture takes more-costly effort to realize.

Under the functional form in (2), we have that $e^*(\theta) = \theta$: The two aforementioned opposing effects of v on the choice of e cancel out and we can identify the choice of effort with the shares allotted to the agent. So, the agent's surplus at the production stage is:

$$\hat{\mathrm{u}}_A(\theta, t, v) := \mathrm{u}_A(e^*(\theta), \theta, t, v) = \frac{1 + \delta\alpha}{2}\theta^2 v - t.$$

3.3 Partnership-constitution contract

Finally, we turn to the problem of designing the contract to allocate θ shares to the agent in exchange for a monetary transfer t. A *partnership-constitution contract* consists of a pair of functions (θ, t) where $\theta : [0, \overline{v}] \to [0, 1]$ denotes a share allocation rule, with $\theta(v)$ denoting the fraction of shares allocated to an agent who reports to be type v, and $t : [0, \overline{v}] \to \mathbb{R}$ represents the transfer or payment rule, with $t(v)$ being the amount an agent who reports type v is charged for his shares.

The principal's expected payoff in an incentive-compatible contract is:

$$u_P(\theta, t) = (1 + \delta\alpha)E\left[(1 - \theta(V))\theta(V)V\right] + E[t(V)],$$

while the agent's expected payoff if he is of type v and reports type $\widetilde{v}$ is:

$$u_A(\widetilde{v}, v) := \hat{\mathrm{u}}_A(\theta(\widetilde{v}), t(\widetilde{v}), v) = \frac{1 + \delta\alpha}{2}\theta(\widetilde{v})^2 v - t(\widetilde{v}).$$

A contract is incentive compatible if and only if the allocation rule $\theta(v)$ is non-decreasing and if truthful payoff for the agent, $U_A(v) := u_A(v, v)$, has the following envelope characterization: $U_A(v) = U_A(0) + \frac{1+\delta\alpha}{2}\int_0^v \theta(\epsilon)^2 \mathrm{d}\epsilon$.

In order to characterize the optimal incentive-compatible contract, we introduce an additional assumption on the distribution of v. Let $\lambda(v)$ be the hazard rate of v, $\lambda(v) = f(v)/(1 - F(v))$, and define the function $\hat{\lambda}(v) := v\lambda(v)$. For every $v \in [0, \overline{v})$, $\hat{\lambda}(v)$ represents v weighted by its hazard.

Assumption 3 (Regularity)**.** *The function $\hat{\lambda}(v)$ is continuous and strictly increasing.*[9]

Assumption 3 is the counterpart to the assumption of monotone virtual utilities in mechanism design under linear utilities. It implies that higher exploitation values will remain higher after weighting them by their hazards. If $\lambda(v)$ is differentiable, the monotonicity condition in Assumption 3 can be expressed as $v\lambda'(v)/\lambda(v) > -1$; for v such that $\lambda'(v) < 0$, this inequality implies that the elasticity of $\lambda(v)$ (in absolute value) is less than 1. A sufficient condition for Assumption 3 is for $\lambda(v)$ itself to be continuous and strictly increasing.

The next proposition characterizes the optimal partnership contract.

Proposition 2 (Optimal constitution)**.** *Under Assumption 3, the optimal partnership-constitution contract, (θ^*, t^*), consists of the share allocation rule:*

$$\theta^*(v) = \frac{\hat{\lambda}(v)}{\hat{\lambda}(v) + 1} \tag{3}$$

and the payment rule $t^*(v) = \frac{1+\delta\alpha}{2}\left(\theta^*(v)^2 v - \int_0^v \theta^*(\epsilon)^2 \mathrm{d}\epsilon\right)$. The resulting partnership is dissolved by means of a Texas shootout where the agent is the designated proposer.

Notice that the expression $\frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1}$ is always between 0 and 1. With the asset being worthless unless exploited, the principal finds it worthwhile to partner with any type v such that $\hat{\lambda}(v) > 0$. At the same time, provided that $\lim_{v \to \overline{v}} \hat{\lambda}(v) < +\infty$, the principal will always want to keep some shares for herself rather than ever selling the business to the agent. If v were public, the principal would always want to sell the asset to the agent; she would choose $\theta^*(v) = 1$ for all v. In the presence of asymmetric information, however, the principal can save on information rents by withholding some of the shares for herself, possibly losing some revenue from the agent's initial payment $t^*(v)$ but capturing a fraction of v directly.

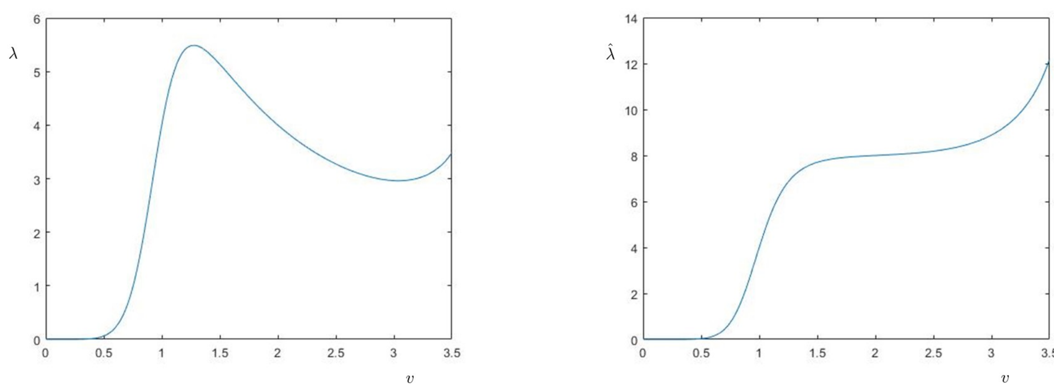
[9] While the weaker assumption that $\hat{\lambda}(v)$ is non-decreasing suffices, the stronger Assumption 3 allows for a simpler characterization of some of the contracts below.

Example 1. Assume that the exploitation value v is uniformly drawn from $[0, 1]$, while the dissolution value w is uniformly drawn from the interval $[0, ev]$. We have $\phi(ev) = \frac{ev}{2}$ and $\hat{\lambda}(v) = \frac{v}{1-v}$. The optimal share allocation rule is $\theta^*(v) = v$, while the optimal transfer rule is given by $t^*(v) = \frac{1}{3}\left(1 + \delta/2\right)v^3$. We can think of the latter as specifying a tailored price $p(v) := \frac{1}{3}\left(1 + \delta/2\right)v^2$ per share: $t^*(v) = p(v)\theta^*(v)$. △

Example 2. Continue to assume that v is drawn uniformly from $[0, 1]$, but imagine that w is now drawn uniformly from the interval $[0, 2ev]$. Here, $\phi(ev) = ev$. The optimal allocation of shares is the same as in Example 1, and the optimal transfer rule has the same structure as that of Example 1; however, the personalized price is now $p(v) = \frac{1}{3}(1 + \delta)v^2$. △

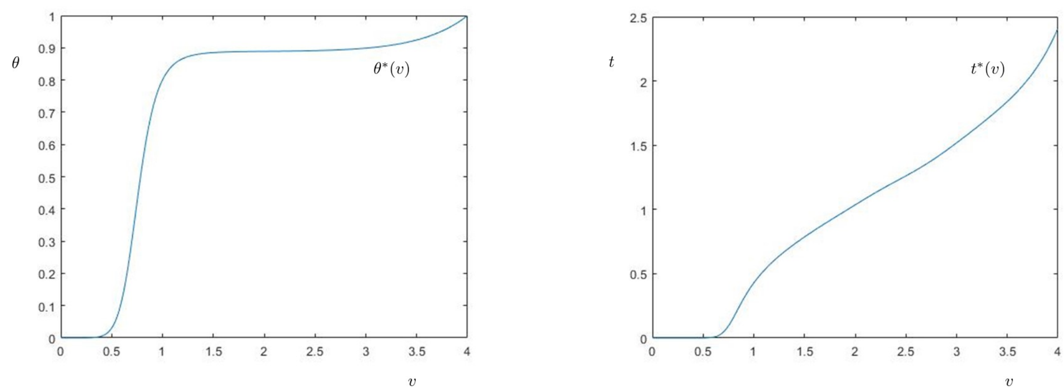
Example 3. Consider the same conditional distribution for w as in Example 1, so that $\phi(ev) = \frac{ev}{2}$. Let v be now drawn from the log-logistic distribution on $[0, 4]$ with scale parameter 1 and shape parameter 8. Figure 2 depicts the functions $\lambda(v)$ and $\hat{\lambda}(v)$ for this distribution. Notice that the hazard rate $\lambda(v)$ is non-monotonic; however, the corresponding $\hat{\lambda}(v)$ is strictly increasing (and continuous), and so Assumption 3 holds. Figure 3 depicts the corresponding optimal allocation rule and the associated payment rule for $\delta = 0.9$. △

If Assumption 3 fails, then the share allocation rule $\theta^*(v)$ in (3) is non-monotonic, hence not incentive compatible. In this situation, the optimal incentive-compatible contract calls for *ironing* the share allocation rule. However, our agent's utility is non-linear in the allocation of shares; as a result, the standard ironing technique of Myerson (1981) does not apply.



(a) $\lambda(v)$ (b) $\hat{\lambda}(v)$

Figure 2: Hazard rates for the log-logistic distribution in Example 3



(a) $\theta^*(v)$ (b) $t^*(v)$ for $\delta = 0.9$

Figure 3: Optimal allocation and transfer rules for $\delta = 0.9$ in Example 3

Fortunately, the integrand in the principal's objective function for implementable contracts, namely the pointwise aggregate surplus net of the agent's information rent, is concave in the allocation of shares; see (A1) in the appendix. Therefore, we can apply the generalized ironing technique of Toikka (2011). Intuitively, this technique entails applying Myerson's approach for each fixed θ and then maximizing to solve for the optimal ironed allocation rule.

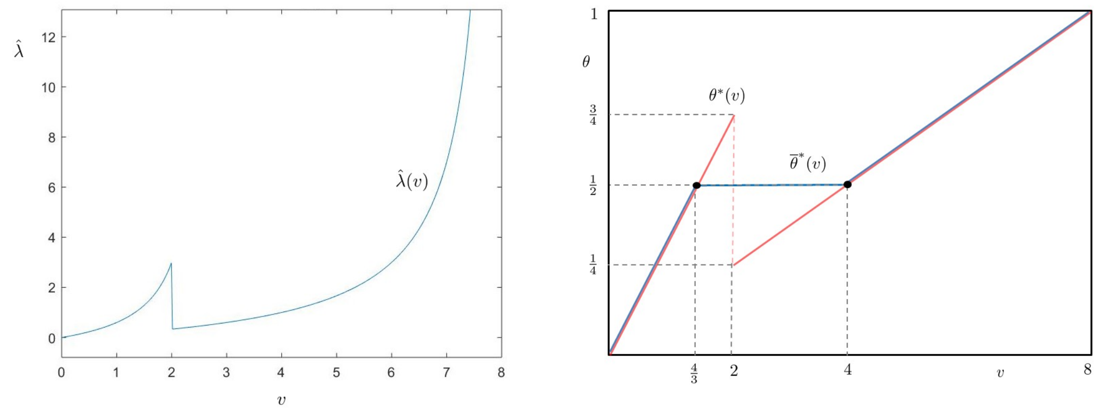
Example 4 (Ironing)**.** Assume that v is drawn from the interval $[0, 8]$ according to the following density function: $f(v) = \frac{3}{8}$ for $v \in [0, 2)$ and $f(v) = \frac{1}{24}$ for $v \in [2, 8]$. Under this distribution, Assumption 3 is violated:

$$\hat{\lambda}(v) = \begin{cases} \frac{3v}{8-3v} & \text{if } v \in [0, 2), \\ \frac{v}{8-v} & \text{if } v \in [2, 8]; \end{cases}$$

see Figure 4a. As a result, the allocation rule $\theta^*(v)$ in (3),

$$\theta^*(v) = \begin{cases} \frac{3v}{8} & \text{if } v \in [0, 2), \\ \frac{v}{8} & \text{if } v \in [2, 8], \end{cases}$$

is infeasible: Types slightly above 2, by misrepresenting themselves as types slightly below 2, can practically triple the amount of shares awarded. Following Toikka (2011), the optimal feasible share allocation rule, call it $\overline{\theta}^*(v)$, is $\overline{\theta}^*(v) = \theta^*(v)$ for $v < \frac{4}{3}$ and $v \geq 4$ but $\overline{\theta}^*(v) = \frac{1}{2}$ for $\frac{4}{3} \leq v < 4$; see Figure 4b. △



(a) $\hat{\lambda}(v)$

(b) $\theta^*(v)$ (red) and $\overline{\theta}^*(v)$ (blue)

Figure 4: Function $\hat{\lambda}(v)$—panel (a)—and comparison between infeasible and ironed optimal allocation rules—panel (b)—in Example 4

3.4 Partnering versus immediate reselling

Under (1), the asset is worthless without the agent's intervention: $\phi(0) = 0$. Thus, the principal will always want to partner with the agent regardless of his type—except perhaps for type $v = 0$. In this subsection, we let the asset have a positive baseline value by replacing (1) with $\phi(ev) = \alpha ev + \beta$ where $\phi(0) = \beta > 0$ represents the revenue that the principal can obtain if she sells the asset immediately without partnering with the agent. This constitutes her outside option.

With a positive outside option, the principal may choose to resell the asset right away without partnering with the agent: The latter raises the asset's value but must be compensated for his costly effort and his private information; if his type is too low, it may not be worthwhile. Thus, we augment the allocation in the contract by $q \in \{0, 1\}$, where $q = 1$ denotes partnering with the agent and $q = 0$ denotes reselling immediately. An augmented partnership-constitution contract is now a triple of functions (q, θ, t) for some $q : [0, \overline{v}] \to \{0, 1\}$.

The principal's expected payoff in an incentive-compatible contract is:

$$u_P(q, \theta, t) = \beta + E\left\{q(V)\left[(1+\delta\alpha)(1-\theta(V))\theta(V)V - (1-\delta(1-\theta(V)))\beta\right]\right\} + E[t(V)],$$

while the agent's expected payoff, defining $\hat{\theta}(v) := q(v)\theta(v)$, is:

$$u_A(\widetilde{v}, v) = \frac{1+\delta\alpha}{2}\hat{\theta}(\widetilde{v})^2 v + \delta\hat{\theta}(\widetilde{v})\beta - t(\widetilde{v}).$$

Incentive compatibility is ensured if and only if $\hat{\theta}(v)$ is non-decreasing and truthful payoff has the structure $U_A(v) = U_A(0) + \frac{1+\delta\alpha}{2}\int_0^v \hat{\theta}(\epsilon)^2 \mathrm{d}\epsilon$.

The next proposition characterizes the optimal partnership contract when the principal may want to resell immediately.

Proposition 3 (Partnering vs. reselling)**.** *Assume that $\phi(ev) = \alpha ev + \beta$ for $\alpha, \beta > 0$. If $\beta \leq \frac{1+\delta\alpha}{2(1-\delta)}\overline{v}$, under Assumption 3, the principal wants to partner with any agent of type $v \geq \underline{v}$, where $\underline{v}$ is the unique solution to the equation:*

$$\frac{v\hat{\lambda}(v)}{\hat{\lambda}(v)+1} = \frac{2(1-\delta)\beta}{1+\delta\alpha}.$$

For such types of the agent, the optimal partnership contract is the same as in Proposition 2. For $v < \underline{v}$, or if $\beta > \frac{1+\delta\alpha}{2(1-\delta)}\overline{v}$, the principal resells the asset immediately.

Example 2 (Revisited)**.** Let v be drawn uniformly from $[0, 1]$, while w is drawn from the uniform distribution on $[1, 1+2ev]$ given ev. Thus, we have $\phi(ev) = ev + 1$. With $\alpha = \beta = 1$, the inequality on β in Proposition 3 becomes $\delta \geq \frac{1}{3}$. If this condition holds, the principal only partners with agents of type $v \geq 2\sqrt{(1-\delta)/(2+\delta)}$. For all other types, she prefers to sell immediately. Notice that this cutoff is strictly decreasing in δ; the more patient the principal (and the agent), the more likely she is to form the partnership. △

4 The Principal as the Proposer

The agent might be able to benefit from his savvy by collecting information rents if the principal is the one designated to propose the price at the dissolution stage. In this section, we explore a scenario where the principal is the proposer in the dissolution proceedings, rather than the agent being put in a position to give away his private expertise.

The structure of the optimal contract where the principal is the proposer depends crucially on her commitment power due to the correlation between the value signals. If the agent reports v truthfully during the constitution negotiations, the principal will be tempted to use this information to fine-tune her beliefs about w and set a dissolution price accordingly. However, this undermines the agent's incentive to be truthful in the first place: If the agent anticipates that the principal will believe his report and use it to set the future price per share, he will have an incentive to misrepresent v to get a better price for himself. Thus, we have a *ratchet effect* type of situation, a situation that can be averted if the principal has the power to commit to a price per share for the dissolution negotiation ex-ante.

In this section, we avoid this complication by looking at the contracting problem when the principal commits to a *fixed* price in the dissolution negotiation. If the principal commits to setting a price $p \geq 0$ per share, when the time comes to dissolve the partnership and after privately observing w, the agent sells his shares if $w < p$ and buys the principal's shares if $w \geq p$.[10] For additional simplicity, we henceforth return to the assumption that $\phi(0) = 0$.

Consider the case where the principal commits to $p = 0$; essentially, she dissolves the partnership by giving her shares away to the agent. Doing so need not be against her best interest if she can extract this surplus back from the agent via the initial payment.[11] The agent's production payoff is:

$$\mathrm{u}_A(e, \theta, t, v) = \theta e v + \delta\alpha e v - t - (1 + \delta\alpha) v \frac{e^2}{2},$$

so $e^*(\theta) = \frac{\theta + \delta\alpha}{1 + \delta\alpha}$. Notice that the agent exerts some effort even if he is given no shares, as he anticipates obtaining the asset for free to resell in the future: $e^*(0) = \frac{\delta\alpha}{1+\delta\alpha} > 0$. In fact, being promised full ownership at no (future) cost induces the agent to always exert more effort than in Proposition 2: $\frac{\theta+\delta\alpha}{1+\delta\alpha} > \theta$ for all $\theta \in [0, 1)$.

The next proposition describes the optimal contract for $p = 0$.

Proposition 4 (Price commitment—$p = 0$)**.** *Under Assumption 3, the optimal partnership-constitution contract when the principal commits ex-ante to dissolution price $p = 0$, (θ_0^*, t_0^*), is as follows. The optimal share allocation rule is:*

$$\theta_0^*(v) = \max\left\{(1 + \delta\alpha)\frac{\hat{\lambda}(v)}{\hat{\lambda}(v) + 1} - \delta\alpha, 0\right\}. \tag{4}$$

For all v such that $\hat{\lambda}(v) \leq \delta\alpha$, the principal awards the agent 0 shares (at no charge). If $\sup\{\hat{\lambda}(v) : v \in [0, \overline{v}]\} > \delta\alpha$, define $\underline{v} := \hat{\lambda}^{-1}(\delta\alpha)$; an agent of type $v \geq \underline{v}$ pays

[10]More generally, the principal could commit ex-ante to a pricing rule $p(v)$, provided that the rule preserves incentive compatibility. The analysis at the end of this section extends to pricing rules that can be ranked in terms of the effort functions they induce.

[11]Proposition 5 at the end of this section shows that, under additional conditions on the distribution of $w|ev$, the price $p = 0$ maximizes the ex-ante aggregate surplus.

$t_0^*(v) = \frac{1}{2(1+\delta\alpha)} \left[(\theta_0^*(v) + \delta\alpha)^2 v - (\delta\alpha)^2 \underline{v} - \int_{\underline{v}}^{v} (\theta_0^*(\epsilon) + \delta\alpha)^2 \mathrm{d}\epsilon \right]$ *for his shares. When the partnership is dissolved, the agent is given full ownership of the asset for free.*

Under the contract in Proposition 4, the principal has no claim to the dissolution value but can charge a higher amount to the agent in the constitution stage. In turn, the agent receives full ownership upon severance, but he exerts more costly effort in exploiting the asset.

The next corollary confirms that, despite this costly extra effort, all types of the agent are better off; the principal, on the other hand, is worse off—the loss in revenue from relinquishing her claim to the dissolution value is not made up by the higher continuation value she can extract from the agent through his initial transfer.

Corollary 1 (Individual value of commitment)**.** *Let $u_P(\theta^*, t^*)$ and $U_A^*(v)$ denote the payoff under the contract in Proposition 2 for the principal and a type-v agent, respectively, and let $u_P(\theta_0^*, t_0^*)$ and $U_{A0}^*(v)$ be the counterpart payoffs under the contract in Proposition 4. We have $u_P(\theta^*, t^*) \geq u_P(\theta_0^*, t_0^*)$ and $U_A^*(v) \leq U_{A0}^*(v)$ for all v.*

Although the principal is worse off, the agent is sufficiently better off that the resulting ex-ante aggregate surplus is higher.

Corollary 2 (Ex-ante social value of commitment)**.** *Denote by $\mathcal{S}$ the ex-ante aggregate surplus under the contract in Proposition 2, $\mathcal{S} = u_P(\theta^*, t^*) + E[U_A^*(V)]$, and let $\mathcal{S}_0$ be its counterpart from Proposition 4, $\mathcal{S}_0 = u_P(\theta_0^*, t_0^*) + E[U_{A0}^*(V)]$. We have that $\mathcal{S} \leq \mathcal{S}_0$.*

Another extreme case, if the support of w is bounded above by some $\overline{w} > 0$, is to set $p = \overline{w}$; the principal guarantees top price for the agent's shares, ensuring that he will always want to sell. However, this alternative should not be appealing to either party: Both the principal and the agent, regardless of his type, are worse off. The agent is guaranteed a fixed dissolution payment, which demotivates effort; to induce more effort, the principal awards him a larger amount of shares. Nonetheless, less value is raised in the partnership and both parties are worse off than in the contract in Proposition 2 in the end.[12]

In characterizing the optimal contract for a parametric $p > 0$, we encounter the following difficulty. The agent's choice of effort will generally depend not only on

[12] A detailed analysis of this case is available in the appendix.

the amount of shares awarded but also on the exploitation value: $e^* = e^*(\theta, v, p)$. In turn, this affects the truncated expectation of w, truncated to the range $w \geq p$ where it is relevant to the agent. (For $w < p$, the agent sells his shares and his payoff is p per share regardless of how low is w.) Thus, characterizing incentive compatibility involves the partial derivative with respect to v of the expression:

$$\begin{aligned} u_A(\widetilde{v}, v, p) &= \theta(\widetilde{v}) e^*(\theta(\widetilde{v}), v, p) v + \delta\theta(\widetilde{v}) p \\ &\quad + \delta E\left[\max\{w - p, 0\} | e^*(\theta(\widetilde{v}), v, p) v\right] - t(\widetilde{v}) - \frac{1 + \delta\alpha}{2} v e^*(\theta(\widetilde{v}), v, p)^2. \end{aligned}$$

In general, the optimal contract is rendered intractable.

The next example features a case where we can fully characterize the optimal contract and look for the price that maximizes ex-ante aggregate surplus.

Example 6. Let $v \sim U[0, 1]$ and let $w|ev$ take on two values, 0 and 1, where the probability that $w = 1$ (and thus, $\phi(ev)$) is ev. Let p be the price at which the principal commits ex-ante; she can always induce the agent to sell his shares for a price of 1, so we can restrict attention to $p \in [0, 1]$. The expected dissolution payoff for the agent is $\theta p + ev(1 - p)$, and his production payoff is:

$$\mathrm{u}_A(e, \theta, t, v) = \theta ev + \delta\theta p + \delta ev(1 - p) - t - (1 + \delta)\, v \frac{e^2}{2}.$$

The optimal choice of effort is then $e^*(\theta) = \frac{\theta + \delta(1-p)}{1+\delta}$, and the agent's truthful surplus in an incentive-compatible contract is:

$$U_A(v) = U_A(0) + \int_0^v \frac{[\theta(\epsilon) + \delta(1 - p)]^2}{2(1 + \delta)} \mathrm{d}\epsilon.$$

The principal's expected revenue (with 0 surplus for the lowest agent type) is:

$$u_P(\theta, t, p) = \int_0^{\overline{v}} \left[[\theta(v) + \delta(1 - p)] v - \frac{[\theta(v) + \delta(1 - p)]^2}{2(1 + \delta)} \right] \mathrm{d}v.$$

Thus, we obtain the optimal share allocation rule:

$$\theta^*(v, p) = \begin{cases} 0 & v < \frac{\delta(1-p)}{1+\delta}; \\ (1 + \delta)v - \delta(1 - p) & \frac{\delta(1-p)}{1+\delta} \leq v \leq \frac{1+\delta(1-p)}{1+\delta}; \\ 1 & v > \frac{1+\delta(1-p)}{1+\delta}. \end{cases}$$

Under this allocation of shares, the highest ex-ante aggregate surplus is attained at $p = 0$; see Figure 5. △

Despite general intractability, under additional assumptions on the distribution of w given ev, we can say that $p = 0$ maximizes the ex-ante aggregate surplus. These additional assumptions include being able to rank the conditional distribution of w in terms of first-order stochastic dominance according to ev.

Proposition 5 (Maximizing ex-ante aggregate surplus)**.** *Assume that w conditional on ev is either: (i) a binary random variable that can take the values 0 or $\overline{w} > 0$, the latter with probability $\frac{ev}{\overline{v}}$; or (ii) a continuous random variable with full-support conditional pdf $g(w|ev)$ and cdf $G(w|ev)$ on $[0, \overline{w}]$ for some $\overline{w} > 0$, with $g(w|\nu)$ continuously differentiable with respect to ν and $G(w|\nu)$ both strictly decreasing and convex with respect to $\nu \in (0, \overline{v})$. Denote by $\mathcal{S}(p)$ the ex-ante aggregate surplus in the optimal contract where the principal commits to setting price $p \geq 0$. We have $\mathcal{S}(p) \leq \mathcal{S}_0$ for all $p \geq 0$.*

Intuitively, giving full ownership to the agent maximizes the amount of effort that he is willing to exert; this, in turn, maximizes aggregate surplus. While intuitive, the proposition is not immediate: p has a negative direct impact on the agent's choice of effort but it also has an indirect impact on effort via the principal's choice of the share allocation rule, whose structure depends on the effort choice in the first place. Formally, while it is straightforward to show that $e_p^{*\prime}(\theta, v, p) \leq 0$, proving

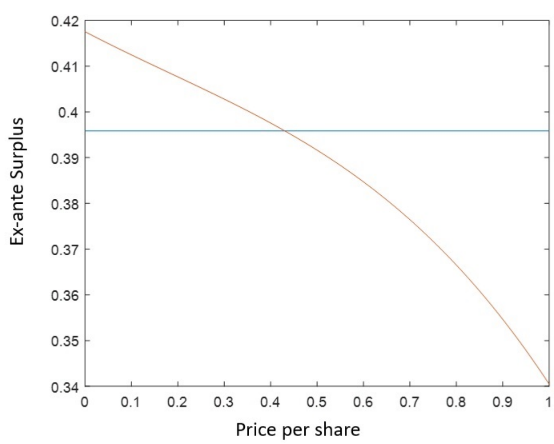


Figure 5: Ex-ante aggregate surplus in Example 6 for $\delta = 0.9$. The blue line represents the ex-ante surplus under Proposition 2, while the red curve represents the ex-ante surplus as a function of p, the fixed price per share.

Proposition 5 entails showing that $e_p^{**\prime}(v,p) \leq 0$, where $e^{**}(v,p) := e^*(\theta^*(v,p),v,p)$ and the shape of $\theta^*(v,p)$ depends on the shape of $e^*(\theta,v,p)$.

Since the contract in Proposition 4 leads to a higher ex-ante aggregate surplus than that of Proposition 2, the agent could approach a trustworthy principal at the ex-ante stage and "bribe" her to commit to $p = 0$; any bribe between $u_P(\theta^*,t^*) - u_P(\theta_0^*,t_0^*)$ and $E[U_{A0}^*(v)] - E[U_A^*(V)]$ is mutually beneficial at the ex-ante stage.

Such proposal, however, breaks down at the interim stage: Low types of the agent enjoy insufficient surplus to compensate the principal. Therefore, if the principal is approached by the agent, she may infer that he is of high type and update the terms of the contract in a way that undermines the agent's incentive to approach her in the first place.

Example 6 (Continued)**.** By committing to $p = 0$, the principal loses $u_P(\theta^*,t^*) - u_P(\theta_0^*,t_0^*) = \frac{\delta^3}{6(1+\delta)^2}$ in expected payoff. For the agent, we have a gain of:

$$U_{A0}^*(v) - U_A^*(v) = \begin{cases} \frac{\delta^2}{2(1+\delta)}v - \frac{1+\delta}{6}v^3 & v < \underline{v}, \\ \frac{\delta^3}{2(1+\delta)^2} - \frac{1+\delta}{6}\underline{v}^3 & v \geq \underline{v}, \end{cases}$$

where $\underline{v} = \frac{\delta}{1+\delta}$. For each δ, there exists a unique $v(\delta) \in \left(0, \frac{\delta}{1+\delta}\right)$ such that $U_{A0}^*(v) - U_A^*(v) < u_P(\theta^*,t^*) - u_P(\theta_0^*,t_0^*)$ for all $v < v(\delta)$; $v(\delta)$ is the solution to the equation $3(1+\delta)\delta^2 v - (1+\delta)^3 v^3 = \delta^3$. The surplus gain of types $v < v(\delta)$ falls short of the loss of the principal.

Assume that there is a cutoff $k \in (0,1)$ such that only types $v \geq k$ approach the principal. She designs the contract in Proposition 2 for types $v \sim U[0,k]$ and the one in Proposition 4 for types $v \sim U[k,1]$. The cutoff type must make just enough extra surplus to compensate the principal. If $k \geq \underline{v}$, the condition:

$$U_{A0}^*(k) - U_A^*(k) = u_P(\theta^*,t^*;k) - u_P(\theta_0^*,t_0^*;k)$$

leads to the cubic equation $k^3 - 2k^2 + 2k - 1 = 0$, whose only real root equals 1. For $k < \underline{v}$, the counterpart of this condition leads to the quadratic equation:

$$(2+3\delta-\delta^3)k^2 - (2+3\delta+6\delta^2-\delta^3)k + (1+\delta)^3 - \delta^3 = 0.$$

However, no such cutoff exists: Regardless of the value of δ, this equation has no real solutions. △

We close this section by empowering the principal to commit ex-ante not only to the dissolution price but also to the allocation rule. Now, the highest ex-ante aggregate surplus can be attained by the principal always selling the asset to the agent at the outset, as she would do if he had no private information.

Corollary 3 (Total commitment power)**.** *If the principal can commit ex-ante to both the dissolution price and the share allocation rule, the ex-ante aggregate surplus is maximized by the principal selling the asset to all types of the agent at the ex-ante stage for a price of* $\frac{1+\delta\alpha}{2}E(V)$*.*

If the asset can only be sold to the agent at the interim stage, the same ex-ante aggregate surplus can be attained but the agent captures *all* the surplus. This is because, awarding full ownership to all types of the agent, the payment cannot be type dependent and must be low enough that even the lowest type is willing to pay it—namely, 0.

5 Extensions

This section discusses the role of the main assumptions in our analysis. In order to simplify the analysis, we assume that the agent takes on the role of proposer in the dissolution stage, as in Section 3.

5.1 General cost function

In this subsection, we present examples with alternative cost of effort functions. Recall that our cost of effort $c(e,v)$ is strictly increasing in both e and v, strictly convex in e, is differentiable with non-negative cross partial derivative, and satisfies: $c(0,v)=0$ for all $v\in[0,\overline{v}]$. The agent's production surplus is:

$$\hat{u}_A(\theta,t,v)=(1+\delta\alpha)\theta e^*(\theta,v)v-c(e^*(\theta,v),v)w-t.$$

Incentive compatibility is equivalent to the conditions that $\theta(v)$ is non-decreasing and truthful payoff has the envelope structure:

$$U_A(v)=\int_0^v \left\{(1+\alpha\delta)\theta(\epsilon)e^*(\theta(\epsilon),\epsilon)+\left[(1+\alpha\delta)\theta(\epsilon)\epsilon-c'_e\left(e^*(\theta(\epsilon),\epsilon),\epsilon\right)\right]e^{*\prime}_v(\theta(\epsilon),\epsilon)\right.$$
$$\left.-\,c'_v(e^*(\theta(\epsilon),\epsilon),\epsilon)\right\}\mathrm{d}\epsilon+U_A(0).$$

The main difficulties in characterizing incentive compatibility and identifying the adequate regularity condition stem from the more involved structure of the integrand and the fact that the sign of $e_v^{*\prime}(\theta(v),v)$ is ambiguous. In general, if monotonicity fails, we can resort to the generalized ironing technique of Toikka (2011) as long as the principal's pointwise payoff is concave in shares. This problem does not arise in the examples below.

Example 7. Consider the same value distributions as in Example 1 and let the cost of effort be given by:

$$c(e,v) = \left(1+\frac{\delta}{2}\right)\frac{e^2}{2}\sqrt{v}.$$

The optimal choice of effort by the agent is $e^*(\theta,v) = \theta\sqrt{v}$, which is always between 0 and 1. His production payoff is:

$$\hat{\mathrm{u}}_A(\theta,t,v) = \frac{2+\delta}{4}\theta^2 v^{\frac{3}{2}} - t,$$

while truthful surplus in an incentive-compatible contract is:

$$U_A(v) = U_A(0) + \frac{3}{8}(2+\delta)\int_0^v \theta(\epsilon)^2\sqrt{\epsilon}\mathrm{d}\epsilon.$$

Here, the optimal share allocation rule is $\theta^*(v) = \frac{8v}{5v+3}$; see Figure 6. △

Example 8. Once again, take the value distributions from Example 1, but now let the cost of effort be $c(e,v) = (1+\frac{\delta}{2})\frac{e+e^2}{2}v$. The optimal choice of effort is $e^*(\theta,v) = \max\{(1+\delta/2)\theta - 1/2, 0\}$, which is always less than 1 but is positive for $(2+\delta)\theta \geq 1$.

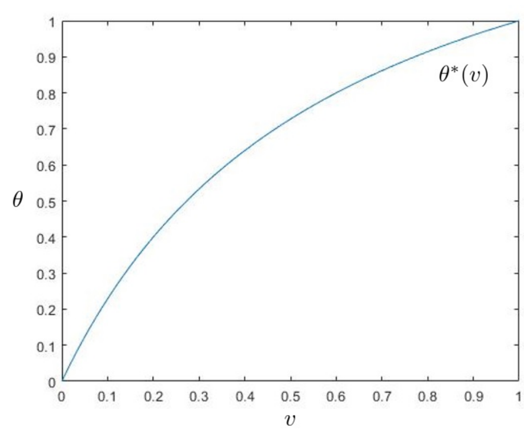


Figure 6: Optimal allocation rule in Example 7.

Focus on the latter case; we have:

$$\hat{u}_A(\theta, t, v) = \left[\left(1 + \frac{\delta}{2}\right)^2 \frac{\theta^2}{2} - \left(1 + \frac{\delta}{2}\right) \frac{\theta}{4} + \frac{1+\delta}{8} \right] v - t,$$

while truthful surplus in an incentive-compatible contract is:

$$U_A(v) = U_A(0) + \frac{(2+\delta)^2}{8} \int_0^v \theta(\epsilon)^2 - \frac{2+\delta}{8} \int_0^v \theta(\epsilon) \mathrm{d}\epsilon + \frac{1+\delta}{8} v.$$

Pointwise maximization of the principal's feasible payoff yields:

$$\theta^*(v) = \frac{(2+\delta)\frac{v}{1-v} + 1}{(2+\delta)\frac{v}{1-v} + 2 + \delta};$$

notice that $(2+\delta)\theta^*(v) \geq 1$ for all v. △

5.2 General correlation structure

If the function $\phi(ev)$ is non-linear but it is differentiable and strictly increasing, we can expand $\phi(ev)$ as $\phi(ev) \approx \phi(0) + \phi'(0)ev$, with $\phi'(0)$ playing the role of α. With this expansion, the contracts in Proposition 2 if $\phi(0) = 0$ or in Proposition 3 if $\phi(0) > 0$ are approximately optimal. Without resorting to this approximation, however, little can be said in general; the problem can become intractable.

Below is a tractable example where $\phi(ev)$ is quadratic.

Example 9. Let v be uniformly drawn from the interval $[0,1]$, and let $\phi(ev) = (ev)^2$. Correspondingly, let the cost of effort be $c(e, v) = (v + \delta\phi(v)) \frac{e^2}{2} = \left(v + \delta v^2\right) \frac{e^2}{2}$. The optimal choice of effort is:

$$e^*(\theta, v) = \frac{\theta}{1 + \delta v(1 - 2\theta)},$$

which is always non-negative but is less than 1 provided $\theta \leq \frac{1+\delta v}{1+2\delta v}$. The principal's expected payoff in an incentive-compatible contract is:

$$u_P(\theta, t) = E\left[\frac{2V(1+\delta V)\theta(V) - (1+3\delta V^2)\theta(V)^2}{2[1+\delta V(1-2\theta(V))]^2} \right].$$

The optimal allocation-rule for shares, $\theta^*(v)$, solves the following quadratic equation:

$$2(1+3\delta v^2)(1-\delta v)\theta^2 + (1+\delta v)(1-4v+5\delta v^2)\theta - v(1+\delta v)^2 = 0$$

on the interval $[0, (1+\delta v)/(1+2\delta v)]$. Figure 7 presents $\theta^*(v)$ for $\delta = 0.9$. For all types above v^* such that $\theta^*(v^*) = \frac{1+\delta v^*}{1+2\delta v^*}$, the agent exerts maximal effort and so the principal need not offer him any more shares. $\triangle$

5.3 Private termination value

While the principal is set to resell the asset if she buys back her partners shares in the dissolution, the agent's savvy may allow him to exploit the asset on his own instead of having to rely on its resale value. To capture this idea, we can assign the agent a (privately-known) private termination value w'.

In the dissolution stage, with the agent calling the price per share, the principal randomizes between buying and selling with probabilities $1-\theta$, θ provided the agent proposes $p = w$. The agent's proposal costlessly discloses the value to the principal, w, not his own private value. Payoffs are $\theta w'$ for the agent and $(1-\theta)w$ for the principal; now, aggregate surplus $w + (w' - w)\theta$ depends on θ.

Thus, allowing for a private dissolution value for the agent means that, ex-post, the size of the aggregate surplus at the dissolution stage—not just how it is split—depends on the ownership structure. Of course, if w and w' have the same

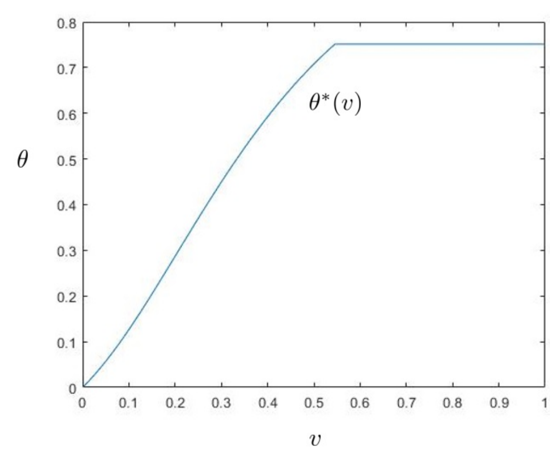


Figure 7: Optimal allocation rule in Example 9 for $\delta = 0.8$. Types above $v^* \approx 0.55$ exert maximal effort, so the principal need not offer these types any further shares beyond $\frac{1+\delta v^*}{1+2\delta v^*} \approx 0.75$.

conditional expectation, then the problem reduces to the common-value problem: $E[W + (W' - W)\theta|ev] = E[W|ev] = \phi(ev)$.

5.4 Long-term partnerships

As discussed in the Introduction (see also footnote 4), the assumption that the partnership lasts for one exploitation period and is then to be dissolved is made for analytical simplicity: It allows us to identify all future negotiations between the partners with the negotiation to dissolve the partnership, and to highlight the role of asymmetric information on the terms of the constitution of the partnership. The dissolution value can be replaced by the proper continuation value in a longer-lived partnership provided that either its duration is definite or that there is an exogenous probability of triggering the event of dissolution.

In a multi-period partnership with new information arriving over time, incentives for truth-telling must be provided every period. If the agent's report is employed as the basis to determine whether the partnership is worth extending for an additional exploitation cycle, the agent may not want to be truthful; instead, he may want to understate the resale value in hopes of buying out the principal for a low price when the partnership is valuable, or to overstate said value in the hope of getting a good price for his shares when the partnership stops being worthwhile.

6 Concluding Remarks

This paper analyzes the problem of a principal contracting with a savvy agent to form a partnership to exploit an asset. The agent has two pieces of valuable private information: (a) How much value they can raise by exploiting the asset; and (b) the asset's resale value, or how much value they can split when dissolving the partnership.

In principle, the agent could use this information in his favor in negotiating with the principal. However, the agent is willing to disclose the resale value of the asset *for free* if the partnership is dissolved by means of a Texas shootout where he is called to propose a price: The principal randomizes between buying and selling in a way that makes his payoff insensitive to the price, so he "might as well" propose a price equal to the resale value. Thus, in the optimal contract, the informational asymmetry regarding the dissolution value does not distort the share allocation rule; the only distortion is due to the information rents from the exploitation value.

If the principal takes on the role of offeror and commits to a dissolution price ex-ante, the partners can attain a higher ex-ante aggregate surplus by granting the agent full ownership for free upon dissolution: The principal earns a lower expected revenue but all types of the agent are sufficiently better off. However, this higher surplus is unattainable at the interim stage because low types of the agent, whose surplus gain is small, are unwilling to fully compensate the principal for her loss of revenue.

The model can be adjusted to accommodate multi-period partnerships provided that either their duration is definite or that the event of dissolving the partnership is otherwise triggered exogenously. However, if the partnership lasts indefinitely and the decision whether to continue or to dissolve the partnership is made based on the agent's current private information, the dissolution negotiation becomes an optimal stopping problem within the partnership-constitution problem. This is the subject of ongoing research.

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A Appendix

A.1 Main Proofs

Proof of Proposition 1. In the dissolution mechanism, allocations are represented by pairs $\kappa = (\kappa_P, \kappa_A) \in \{0,1\}^2$ such that $\kappa_P + \kappa_A = 1$, where $\kappa_P = 1$ represents the principal gaining full ownership, and $\kappa_A = 1$ represents the agent taking over. The payment from the agent to the principal, which may be negative, is $\tau \in \mathbb{R}$. In the first-best scenario where both parties observe w, payoffs are $u_P(w) = \kappa_P w + \tau$ for the principal and $u_A(w) = \kappa_A w - \tau$ for the agent, who has an outside option of $u_A^0(w) = \theta w$. From the agent's participation constraint $u_A(w) \geq u_A^0(w)$, we get that $\tau \leq (\kappa_A - \theta)\, w$. Thus, the principal's first-best payoff is bounded above by $\kappa_P w + (\kappa_A - \theta)\, w = (1-\theta)w$, awarding the agent no surplus. This upper bound is attained in the second-best environment in the Texas-shootout equilibrium described in Proposition 1 of Brooks et al. (2010) with the agent as the proposer. □

Proof of Proposition 2. In any incentive-compatible mechanism, the feasible payoff for the principal is:

$$u_P(\theta, t) = \frac{1+\delta\alpha}{2} \int_0^{\overline{v}} \left[2\theta(v)v - \left(v + \frac{1}{\lambda(v)} \right) \theta(v)^2 \right] f(v)\mathrm{d}v - U_A(0).$$

In the optimal contract, $U_A(0) = 0$. Ignoring the monotonicity constraint, we search for the optimal share allocation rule by pointwise maximization. We look for the maximizer of the following strictly-concave parametric function:

$$J(\theta; v) := 2v\theta - \left(v + \frac{1}{\lambda(v)} \right) \theta^2 = 2v\theta - \left(\frac{v\lambda(v)+1}{\lambda(v)} \right) \theta^2 \qquad \text{(A1)}$$

subject to the constraint $0 \leq \theta \leq 1$. For each $v \in [0, \overline{v}]$, the maximizer of (A1) is $\theta^*(v)$ in (3). Under Assumption 3, $\theta^*(v)$ is non-decreasing. Therefore, the transfer rule $t^*(v)$ ensures incentive compatibility and individual rationality. □

Proof of Proposition 3. In any incentive-compatible mechanism with $U_A(0) = 0$, the feasible payoff for the principal is now:

$$\begin{aligned} u_P(q,\theta,t) &= \beta + \int_0^{\overline{v}} q(v) \left\{ \frac{1+\delta\alpha}{2} \left[2\theta(v)v - \left(v + \frac{1}{\lambda(v)} \right) \theta(v)^2 \right] - (1-\delta)\beta \right\} f(v)\mathrm{d}v \\ &= \beta + \int_0^{\overline{v}} q(v) \left\{ \frac{1+\delta\alpha}{2} J(\theta(v); v) - (1-\delta)\beta \right\} f(v)\mathrm{d}v. \end{aligned}$$

Taking $\theta^*(v)$ from (3), we have:

$$u_P(q,\theta,t) \leq \beta + \int_0^{\overline{v}} \max \left\{ \frac{1+\delta\alpha}{2} v\theta^*(v) - (1-\delta)\beta, 0 \right\} f(v)\mathrm{d}v.$$

Under Assumption 3, $\theta^*(v)$ is continuous and strictly increasing, and so is $\psi(v) := v\theta^*(v)$. Thus, $\psi(v)$ is invertible. Moreover, under the assumption on β, $\frac{2(1-\delta)\beta}{1+\delta\alpha}$ is in the range of $\psi(v)$, so we can define $\underline{v} := \psi^{-1}\left(2(1-\delta)\beta/(1+\delta\alpha)\right)$. This gives:

$$u_P(q,\theta,t) \leq \beta + \int_{\underline{v}}^{\overline{v}} \left[\frac{1+\delta\alpha}{2} v\theta^*(v) - (1-\delta)\beta \right] f(v)\mathrm{d}v.$$

This upper bound is attained by $q^*(v) = I(v \geq \underline{v})$, which is non-decreasing. Thus, $\hat{\theta}^*(v) = q^*(v)\theta^*(v)$ is also non-decreasing under Assumption 3. The transfer rule $t^*(v)$ from Proposition 2 for $v \geq \underline{v}$ guarantees implementability. □

Proof of Proposition 4. In an incentive-compatible mechanism with $U_A(0) = 0$,

$$u_P(\theta,t) = E\left[(\theta(V) + \delta\alpha)V - \frac{1}{1+\delta\alpha} \left(V + \frac{1}{\lambda(V)} \right) \frac{(\theta(V)+\delta\alpha)^2}{2} \right].$$

We now look for the maximizer of the following strictly-concave parametric function:

$$K(\theta; v) = (\theta + \delta\alpha)v - \frac{1}{1+\delta\alpha} \left(v + \frac{1}{\lambda(v)} \right) \frac{(\theta+\delta\alpha)^2}{2}. \tag{A2}$$

This function is maximized at $\theta_0^*(v)$ in (4), which is continuous and non-decreasing under Assumption 3. The corresponding transfer rule in this contract is $\tau^*(v) := \frac{1}{2(1+\delta\alpha)}\left[(\theta_0^*(v)+\delta\alpha)^2 v - \int_0^v (\theta_0^*(\epsilon)+\delta\alpha)^2 \mathrm{d}\epsilon \right]$. For all v such that $\hat{\lambda}(v) \leq \delta\alpha$, we have $\theta_0^*(v) = 0$ and $\tau^*(v) = \frac{1}{2(1+\delta\alpha)}\left[(\delta\alpha)^2 v - \int_0^v (\delta\alpha)^2 \mathrm{d}\epsilon \right] = 0$. For all $v \geq \underline{v} := \hat{\lambda}^{-1}(\delta\alpha)$, if any, we have:

$$\tau^*(v) = \frac{1}{2(1+\delta\alpha)} \left[(\theta_0^*(v)+\delta\alpha)^2 v - \int_0^{\underline{v}} (\theta_0^*(\epsilon)+\delta\alpha)^2 \mathrm{d}\epsilon - \int_{\underline{v}}^{v} (\theta_0^*(\epsilon)+\delta\alpha)^2 \mathrm{d}\epsilon \right]$$

$$= \frac{1}{2(1+\delta\alpha)} \left[(\theta_0^*(v) + \delta\alpha)^2 v - (\delta\alpha)^2 \underline{v} - \int_{\underline{v}}^{v} (\theta_0^*(\epsilon) + \delta\alpha)^2 \mathrm{d}\epsilon \right] =: t_0^*(v).$$

This establishes the proposition. □

Proof of Corollary 1. For the principal,

$$u_P(\theta_0^*, t_0^*) = \frac{1+\delta\alpha}{2} E\left[I(V \leq \underline{v}) J\left(\frac{\delta\alpha}{\delta\alpha+1}; V\right) + I(V > \underline{v}) J\left(\frac{\hat{\lambda}(V)}{\hat{\lambda}(V)+1}; V\right)\right]$$
$$\leq \frac{1+\delta\alpha}{2} E\left[J\left(\frac{\hat{\lambda}(V)}{\hat{\lambda}(V)+1}; V\right)\right] = u_P(\theta^*, t^*);$$

the inequality follows from the fact that $\frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1}$ maximizes $J(\theta; v)$. For the agent of type $v \leq \underline{v}$, we have:

$$U_{A0}^*(v) = \frac{1+\delta\alpha}{2} \int_0^v \left(\frac{\delta\alpha}{1+\delta\alpha}\right)^2 \mathrm{d}\epsilon \geq \frac{1+\delta\alpha}{2} \int_0^v \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1}\right)^2 \mathrm{d}\epsilon = U_A^*(v).$$

Finally, for $v > \underline{v}$ (if any),

$$U_{A0}^*(v) = \frac{1+\delta\alpha}{2} \int_0^{\underline{v}} \left(\frac{\delta\alpha}{1+\delta\alpha}\right)^2 \mathrm{d}\epsilon + \frac{1+\delta\alpha}{2} \int_{\underline{v}}^{v} \left(\frac{\theta_0^*(\epsilon)+\delta\alpha}{1+\delta\alpha}\right)^2 \mathrm{d}\epsilon$$
$$= \frac{1+\delta\alpha}{2} \int_0^{\underline{v}} \left(\frac{\delta\alpha}{1+\delta\alpha}\right)^2 \mathrm{d}\epsilon + \frac{1+\delta\alpha}{2} \int_{\underline{v}}^{v} \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1}\right)^2 \mathrm{d}\epsilon$$
$$\geq \frac{1+\delta\alpha}{2} \int_0^{v} \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1}\right)^2 \mathrm{d}\epsilon = U_A^*(v).$$

This establishes the result. □

Proof of Corollary 2. We can write the ex-ante aggregate surplus under a contract with effort-choice $\widetilde{e}(\theta, v)$ and share allocation rule $\widetilde{\theta}(v)$, $\widetilde{S}$, as:

$$\widetilde{\mathcal{S}} = \frac{1+\delta\alpha}{2} \int_0^{\overline{v}} \left[2\widetilde{e}\left(\widetilde{\theta}(v), v\right) - \widetilde{e}\left(\widetilde{\theta}(v), v\right)^2 \right] v f(v) \mathrm{d}v. \tag{A3}$$

For Proposition 2, (A3) is:

$$\mathcal{S} = \frac{1+\delta\alpha}{2} \int_0^{\overline{v}} \left[\frac{2\hat{\lambda}(v)}{\hat{\lambda}(v)+1} - \left(\frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1}\right)^2 \right] v f(v) \mathrm{d}v.$$

Compare this surplus with the surplus under Proposition 4:

$$\int_0^{\underline{v}} \left[\left(\frac{2\delta\alpha}{\delta\alpha+1} \right) - \left(\frac{\delta\alpha}{\delta\alpha+1} \right)^2 \right] v f(v) \mathrm{d}v + \int_{\underline{v}}^{\overline{v}} \left[\frac{2\hat{\lambda}(v)}{\hat{\lambda}(v)+1} - \left(\frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1} \right)^2 \right] v f(v) \mathrm{d}v$$

$$\geq \int_0^{\overline{v}} \left[\frac{2\hat{\lambda}(v)}{\hat{\lambda}(v)+1} - \left(\frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1} \right)^2 \right] v f(v) \mathrm{d}v,$$

where the inequality follows from the fact that $\hat{\lambda}(v)/(\hat{\lambda}(v)+1) \leq \delta\alpha/(\delta\alpha+1)$ for for $v \leq \underline{v}$ and that the function $h(x) = 2x - x^2$ is strictly increasing on $[0,1)$. It follows that $\mathcal{S}_0 \geq \mathcal{S}$. □

Proof of Proposition 5. We prove this proposition by showing that the effort exerted within the partnership, $e^{**}(v,p) = e^*(\theta^*(v,p),v,p)$, is non-increasing in p; the result then follows from the fact the integrand in (A3), $2e - e^2$, is strictly increasing on $[0,1)$. The proof for part (i) is a straightforward extension of Example 6; details are available from the author upon request. Here, we focus on part (ii).

Since $w \leq \overline{w}$, the principal can focus on $p \leq \overline{w}$; furthermore, we can rule out $p = \overline{w}$, as it leads to a lower ex-ante aggregate surplus than in Proposition 2. Thus, fix $p \in [0,\overline{w})$. If the exploitation value is null, the agent might as well exert no effort. Otherwise, for every $v \in (0,\overline{v})$, the agent chooses effort to maximize the function:

$$\mathrm{u}_A(e,p,\theta,t,v) = \theta e v + \delta \left[\overline{w} - (1-\theta)p - \int_p^{\overline{w}} G(w|ev)\mathrm{d}w \right] - t - (1+\delta\alpha)v\frac{e^2}{2}.$$

Under the additional assumptions, this function is strictly concave (as a function of e), so it has a unique maximizer e^*. For each triple (θ,v,p) with $0 \leq p < \overline{w}$ and $0 < v < \overline{v}$, e^* solves the first-order condition:

$$\theta - \delta \int_p^{\overline{w}} \frac{\partial G(w|e^*v)}{\partial \nu}\mathrm{d}w - (1+\delta\alpha)e^* = 0. \tag{A4}$$

Thus, it has the following properties:

$$\frac{\partial e^*(\theta,v,p)}{\partial\theta} = \frac{1}{\delta v \int_p^{\overline{w}} \frac{\partial^2 G(w|ev)}{\partial \nu^2}\mathrm{d}w + (1+\delta\alpha)} > 0; \tag{A5}$$

$$\frac{\partial e^*(\theta,v,p)}{\partial v} = -\frac{\delta e^*(\theta,v,p) \int_p^{\overline{w}} \frac{\partial^2 G(w|e^*(\theta,v,p)v)}{\partial \nu^2}\mathrm{d}w}{\delta v \int_p^{\overline{w}} \frac{\partial^2 G(w|ev)}{\partial \nu^2}\mathrm{d}w + (1+\delta\alpha)} \leq 0; \tag{A6}$$

$$\frac{\partial e^*(\theta,v,p)}{\partial p} = \frac{\delta \frac{\partial G(p|e^*(\theta,v,p)v)}{\partial \nu}}{\delta v \int_p^{\overline{w}} \frac{\partial^2 G(w|ev)}{\partial \nu^2} \mathrm{d}w + (1+\delta\alpha)} \leq 0. \tag{A7}$$

We have:

$$\begin{aligned}\frac{\partial \hat{\mathrm{u}}_A(\theta,t,v,p)}{\partial v} &= e^*(\theta,v,p)\left[\theta - \int_p^{\overline{w}} \frac{\partial G(w|e^* v)}{\partial \nu}\mathrm{d}w - \frac{1+\delta\alpha}{2} e^*(\theta,v,p)\right] \\ &= \frac{1+\delta\alpha}{2} e^*(\theta,v,p)^2,\end{aligned}$$

where the last equality follows from (A4). Hence, truthful surplus in a contract with $U_A(0) = 0$ is $U_A(v) = \frac{1+\delta\alpha}{2}\int_0^v e^*(\theta(\epsilon,p),\epsilon,p)^2 \mathrm{d}\epsilon$; from the point of view of the ex-ante stage, this is:

$$E[U_A(V)] = \frac{1+\delta\alpha}{2}\int_0^{\overline{v}} \frac{e^*(\theta(v,p),v,p)^2}{\lambda(v)} f(v)\mathrm{d}v.$$

The principal's payoff in an incentive-compatible contract satisfies:

$$u_P(\theta,t,p) \propto E\left\{\frac{1}{\lambda(V)}\left[2e^*(\theta(V,p),V,p)\hat{\lambda}(V) - \left(\hat{\lambda}(V)+1\right)e^*(\theta(V,p),V,p)^2\right]\right\}.$$

Consider the function:

$$R(\theta,v,p) := 2e^*(\theta,v,p)\hat{\lambda}(v) - \left(\hat{\lambda}(v)+1\right)e^*(\theta,v,p)^2.$$

Ignoring the monotonicity constraint, the principal determines the allocation rule $\theta^*(v,p)$ by maximizing $R(\theta,v,p)$ with respect to θ. Under Assumption 3, (A5) and (A6) imply that $\theta^*(v,p)$ is non-decreasing in v, hence it is implementable. Now, recall $e^{**}(v,p) = e^*(\theta^*(v,p),v,p)$. For types v such that $\theta(v,p) = 0$, we have that $e^{**}(v,p) = e^*(0,v,p)$, which is non-increasing in p. The same is true for all v such that $\theta(v,p) = 1$. Finally, for all v such that $\theta^*(v,p) \in (0,1)$, we have $e^{**}(v,p) = \frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1}$; here, p does not affect the equilibrium effort whatsoever. It follows that $e^{**}(v,p)$ is non-increasing in p for every v. □

Proof of Corollary 3. Being committed to $p = 0$, the agent's choice of effort is $e^*(\theta) = \frac{\theta+\delta\alpha}{1+\delta\alpha}$, and the ex-ante aggregate surplus is:

$$\mathcal{S}_0(\theta) := \frac{1+\delta\alpha}{2}\int_0^{\overline{v}} \left[2\frac{\theta(v)+\delta}{1+\delta} - \left(\frac{\theta(v)+\delta}{1+\delta}\right)^2\right] v f(v)\mathrm{d}v.$$

The term in square brackets is bounded above by 1. This bound is attained by setting $\theta(v) = 1$ for all $v \in [0, \overline{v}]$. To ensure ex-ante participation, the payment must be no grater than $\frac{1+\delta\alpha}{2}E(V)$. $\square$

A.2 Additional computations for Example 4

In Example 4, the cdf and hazard rate for v are:

$$F(v) = \begin{cases} \frac{3}{8}v & 0 \le v < 2, \\ \frac{2}{3} + \frac{v}{24} & 2 \le v \le 8; \end{cases} \quad \lambda(v) = \begin{cases} \frac{3}{8-3v} & 0 \le v < 2, \\ \frac{1}{8-v} & 2 \le v \le 8; \end{cases}$$

and the function $J(\theta, v)$ from (A1) is:

$$J(\theta, v) = \begin{cases} 2v\theta - \frac{8}{3}\theta^2 & 0 \le v < 2, \\ 2v\theta - 8\theta^2 & 2 \le v \le 8. \end{cases}$$

Following Toikka (2011), we perform a change of variable on v to obtain a uniformly distributed type. Take the quantile function $v = F^{-1}(p)$, which is given by:

$$F^{-1}(p) = \begin{cases} \frac{3}{8}p & 0 \le p < \frac{3}{4}, \\ 24p - 16 & \frac{3}{4} \le p \le 1, \end{cases}$$

and define $\widetilde{J}(\theta, p) := J(\theta; F^{-1}(p))$; we have:

$$\widetilde{J}(\theta, p) = \begin{cases} \frac{16}{3}p\theta - \frac{8}{3}\theta^2 & 0 \le p < \frac{3}{4}, \\ 16(3p-2)\theta - 8\theta^2 & \frac{3}{4} \le p \le 1. \end{cases}$$

First, we take the (piecewise) partial derivative of $\widetilde{J}(\theta, p)$ with respect to θ:

$$\widetilde{J}'_\theta(\theta, p) = \begin{cases} \frac{16}{3}(p - \theta) & 0 < p < \frac{3}{4}, \\ 48p - 32 - 16\theta & \frac{3}{4} < p \le 1. \end{cases}$$

Next, compute $H(\theta, p) := \int_0^p \widetilde{J}'_\theta(\theta, r)\mathrm{d}r$:

$$H(\theta, p) = \begin{cases} \frac{8}{3}p^2 - \frac{16}{3}\theta p & 0 \le p < \frac{3}{4}, \\ 12 + 8\theta - 16(2+\theta)p + 24p^2 & \frac{3}{4} \le p \le 1. \end{cases}$$

For each fixed θ, both pieces of $H(\theta, p)$ are parabolas in p. Thus, its convex hull, $\overline{H}(\theta, p)$, is obtained by "patching" $H(\theta, p)$ with a linear (possibly flat) function; there are two values $p_1 < \frac{3}{4}$ and $p_2 \geq \frac{3}{4}$ for p and two parameters $a, b \in \mathbb{R}$ such that:

$$\overline{H}(\theta, p) = \begin{cases} \frac{8}{3}p^2 - \frac{16}{3}\theta p & 0 \leq p < p_1, \\ ap + b & p_1 \leq p < p_2, \\ 12 + 8\theta - 16(2+\theta)p + 24p^2 & p_2 \leq p \leq 1. \end{cases}$$

Now, take the (piecewise) partial derivative of $\overline{H}(\theta, p)$ with respect to p; call it $\overline{h}(\theta, p)$:

$$\overline{h}(\theta, p) := \overline{H}'_p(\theta, p) = \begin{cases} \frac{16}{3}(p - \theta) & 0 < p < p_1, \\ a & p_1 \leq p \leq p_2, \\ 24p - 16(2+\theta) & p_2 < p \leq 1. \end{cases}$$

Integrate $\overline{h}(\theta, p)$ with respect to θ to obtained the "ironed" objective function for the principal, $\overline{J}(\theta, p)$:

$$\overline{J}(\theta, p) := \int_0^\theta \overline{h}(s, p) \mathrm{d}s = \begin{cases} \frac{16}{3}p\theta - \frac{8}{3}\theta^2 & 0 \leq p < p_1, \\ a\theta & p_1 \leq p < p_2, \\ 16(3p - 2)\theta - 8\theta^2 & p_2 \leq p \leq 1. \end{cases}$$

In order to identify a, b, p_1, and p_2, notice that the linear "patch" on $H(\theta, p)$ must paste smoothly with its two pieces:

$$\frac{16}{3}(p_1 - \theta) = a = 48p_2 - 16(2+\theta); \quad \text{(smooth pasting)} \tag{A8}$$

$$\frac{8}{3}p_1^2 - \frac{16}{3}\theta p_1 = ap_1 + b; \quad \text{(value matching at } p_1\text{)} \tag{A9}$$

$$24p_2^2 - 16(2+\theta)p_2 + 4(3+2\theta) = ap_2 + b. \quad \text{(value matching at } p_2\text{)} \tag{A10}$$

From (1), we get $p_1 = 9p_2 - 6 - 2\theta$. From (1) and (3), $b = -24p_2^2 + 4(3+2\theta)$. Thus, (2) becomes $\frac{16}{3}p_1^2 = -24p_2^2 + 4(3+2\theta)$. Combining the latter equation with (1) yields the quadratic equation $144p_2^2 - 72(3+\theta)p_2 + 81 + 27\theta + 8\theta^2 = 0$. At the same time, we want to maximize the third piece of $\overline{J}(\theta, p)$ with respect to θ. (Notice that the boundaries for each piece, p_1 and p_2, may in principle depend on θ itself.) For $p \in (0, p_1)$, the maximizer is $\theta(p) = p$; for $p \in [p_2, 1)$, we have $\theta(p) = 3p - 2$—recall that $p_2 \geq \frac{3}{4} > \frac{2}{3}$. Plugging the latter expression evaluated at p_2 in the quadratic equation above yields:

$$144p_2^2 - 72(3p_2 + 1)p_2 + 81 + 27(3p_2 - 2) + 8(3p_2 - 2)^2 = 0.$$

Expanding the binomial squared and regrouping terms, this equation reduces to the linear equation $-6p_2 + 5 = 0$, so we get $p_2 = \frac{5}{6}$; thus, $\theta(p_2) = \frac{1}{2}$ and, from (1), $p_1 = \frac{1}{2}$. The optimal ironed allocation rule in terms of p is then:

$$\overline{\theta}^*(p) = \begin{cases} p & p < \frac{1}{2}, \\ \frac{1}{2} & 12 \leq p < \frac{5}{6}, \\ 3p - 2 & p \geq \frac{5}{6}. \end{cases}$$

Changing variables back to v by setting $p = F(v)$ yields $\overline{\theta}^*(v)$ in Example 4.

A.3 Commitment to $p = \overline{w}$

If the principal commits to $p = \overline{w}$, the agent's production payoff is $\mathrm{u}_A(e;\theta,t,v) = \theta ev + \delta\theta\overline{w} - t - (1+\delta\alpha)v\frac{e^2}{2}$, which is maximized at $e^*(\theta) = \frac{\theta}{1+\delta\alpha}$; since the agent has a guaranteed dissolution payoff, he will exert less effort: $e^*(\theta) < \theta$ for all $\theta > 0$. Expected payoffs in a constitution contract are now:

$$u_A(\widetilde{v};v) = \frac{\theta(\widetilde{v})^2}{2(1+\delta\alpha)}v + \delta\theta(\widetilde{v})\overline{w} - t(\widetilde{v});$$
$$u_P(\theta,t) = E\left\{(1-\theta(V))e^*(\theta(V))V + \delta\left[\alpha e^*(\theta(V))V - \theta(V)\overline{w}\right]\right\} + E[t(V)].$$

Proposition A1 (Price commitment—$p = \overline{w}$)**.** *Under Assumption 3, the optimal partnership-constitution contract when the principal commits ex-ante to dissolution price $p = \overline{w}$ is as follows. The optimal share allocation rule is:*

$$\theta_1^*(v) := \min\left\{(1+\delta\alpha)\frac{\hat{\lambda}(v)}{\hat{\lambda}(v)+1}, 1\right\}.$$

Define $\underline{v}$ as $\hat{\lambda}^{-1}\left(\frac{1}{\delta\alpha}\right)$ if $\max\{\hat{\lambda}(v) : v \in [0,\overline{v}]\} > \frac{1}{\delta\alpha}$ and as $\overline{v}$ otherwise. An agent of type $v \leq \underline{v}$ pays $t_1^*(v) := \frac{1}{2(1+\delta\alpha)}\left[\theta_1^*(v)^2 v - \int_0^v \theta_1^*(\epsilon)^2 \mathrm{d}\epsilon\right] + \delta\overline{w}\theta_1^*(v)$, while an agent of type $v > \underline{v}$, if any, is awarded full ownership for $t_{1f}^*(v) := \frac{1}{2(1+\delta\alpha)}\left[\underline{v} - \int_0^{\underline{v}} \theta_1^*(\epsilon)^2 \mathrm{d}\epsilon\right] + \delta\overline{w}$. When the partnership is dissolved, the agent sells his shares back to the principal.

Proof. In an incentive-compatible mechanism, we have:

$$t(v) = \frac{\theta(v)^2}{2(1+\delta\alpha)}v + \delta\overline{w}\theta(v) - \frac{1}{2(1+\delta\alpha)}\int_0^v \theta(\epsilon)^2 \mathrm{d}\epsilon.$$

The principal's payoff is:

$$u_P(\theta, t) = \frac{1}{1+\delta\alpha} E\left[(1+\delta\alpha)\theta(V)V - \left(V + \frac{1}{\lambda(V)}\right)\frac{\theta(V)^2}{2}\right].$$

So, we look for the maximizer of the strictly-concave parametric function:

$$L(\theta; v) = (1+\delta\alpha)v\theta - \left(v + \frac{1}{\lambda(v)}\right)\frac{\theta^2}{2}.$$

The maximizer is $\theta_1^*(v)$, which is non-decreasing under Assumption 3; the transfer function is $\tau^*(v) := \frac{1}{2(1+\delta\alpha)}\left[\theta_1^*(v)^2 v - \int_0^v \theta_1^*(\epsilon)^2 \mathrm{d}\epsilon\right] + \delta\overline{w}\theta_1^*(v)$. For $v \leq \underline{v}$, we have:

$$\tau^*(v) = \frac{1}{2(1+\delta\alpha)}\left[\theta_1^*(v)^2 v - \int_0^v \theta_1^*(\epsilon)^2 \mathrm{d}\epsilon\right] + \delta\overline{w}\theta_1^*(v) =: t_1^*(v);$$

for $v > \underline{v}$, if any such v exists,

$$\begin{aligned}
\tau^*(v) &= \frac{1}{2(1+\delta\alpha)}\left[v - \int_0^{\underline{v}} \theta_1^*(\epsilon)^2 \mathrm{d}\epsilon - \int_{\underline{v}}^v \mathrm{d}\epsilon\right] + \delta\overline{w} \\
&= \frac{1}{2(1+\delta\alpha)}\left[\underline{v} - \int_0^{\underline{v}} \theta_1^*(\epsilon)^2 \mathrm{d}\epsilon\right] + \delta\overline{w} =: t_{1f}^*(v).
\end{aligned}$$

This establishes the proposition. □

Corollary A1 (The value of commitment, II)**.** *Denote by $u_P(\theta_1^*, t_1^*)$ and $U_{A1}^*(v)$ the payoffs for the principal and a type-v agent, respectively, under the contract in the proposition above. We have $u_P(\theta^*, t^*) \geq u_P(\theta_1^*, t_1^*)$ and $U_A^*(v) \geq U_{A1}^*(v)$ for all v.*

Proof. For the principal, we have:

$$\begin{aligned}
u_P(\theta_1^*, t_1^*) &= (1+\delta\alpha)E\left\{I(V < \underline{v})\left[\frac{V\hat{\lambda}(V)}{\hat{\lambda}(V)+1} - \left(V + \frac{1}{\lambda(V)}\right)\frac{1}{2}\left(\frac{\hat{\lambda}(V)}{\hat{\lambda}(V)+1}\right)^2\right]\right\} \\
&\quad + \frac{1}{1+\delta\alpha}E\left\{I(V > \underline{v})\left[(1+\delta\alpha)v - \left(V + \frac{1}{\lambda(V)}\right)\frac{1}{2}\right]\right\} \\
&= (1+\delta\alpha)E\left\{I(V < \underline{v})\left[\frac{V\hat{\lambda}(V)}{\hat{\lambda}(V)+1} - \left(V + \frac{1}{\lambda(V)}\right)\frac{1}{2}\left(\frac{\hat{\lambda}(V)}{\hat{\lambda}(V)+1}\right)^2\right]\right\} \\
&\quad + (1+\delta\alpha)E\left\{I(V > \underline{v})\left[\frac{1}{1+\delta\alpha}v - \left(V + \frac{1}{\lambda(V)}\right)\frac{1}{2}\left(\frac{1}{1+\delta\alpha}\right)^2\right]\right\}
\end{aligned}$$

$$
\begin{aligned}
u_P(\theta_1^*, t_1^*) &= \frac{1+\delta\alpha}{2} E\left[I(V < \underline{v}) J\left(\frac{\hat{\lambda}(V)}{\hat{\lambda}(V)+1}; V \right) + I(V > \underline{v}) J\left(\frac{1}{1+\delta\alpha}; V \right) \right] \\
&\leq \frac{1+\delta\alpha}{2} E\left[J\left(\theta^*(V); V \right) \right] = u_P(\theta^*, t^*),
\end{aligned}
$$

where $J(\theta; v)$ is the function in (A1). For the agent of type $v \leq \underline{v}$,

$$
\begin{aligned}
U_{A1}^*(v) &= \frac{1}{2(1+\delta\alpha)} \int_0^v \left((1+\delta\alpha) \frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1} \right)^2 \mathrm{d}\epsilon \\
&= \frac{1+\delta\alpha}{2} \int_0^v \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1} \right)^2 \mathrm{d}\epsilon = U_A^*(v).
\end{aligned}
$$

Finally, for the agent of type $v > \underline{v}$, if any,

$$
\begin{aligned}
U_{A1}^*(v) &= \frac{1}{2(1+\delta\alpha)} \left[\int_0^{\underline{v}} \left((1+\delta\alpha) \frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1} \right)^2 \mathrm{d}\epsilon + \int_{\underline{v}}^v \mathrm{d}\epsilon \right] \\
&= \frac{1+\delta\alpha}{2} \left[\int_0^{\underline{v}} \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1} \right)^2 \mathrm{d}\epsilon + \left(\frac{1}{1+\delta\alpha} \right)^2 (v - \underline{v}) \right] \\
&\leq \frac{1+\delta\alpha}{2} \left[\int_0^{\underline{v}} \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1} \right)^2 \mathrm{d}\epsilon + \int_{\underline{v}}^v \left(\frac{\hat{\lambda}(\epsilon)}{\hat{\lambda}(\epsilon)+1} \right)^2 \mathrm{d}\epsilon \right] = U_A^*(v).
\end{aligned}
$$

This establishes the result. □