

Algebra Refresher Workshop: Key Skills

1. **F.O.I.L.** \Rightarrow Used when multiplying *binomials*, expressions with two terms such as

$$(a + b)(c + d)$$

First two terms: $(\mathbf{a} + b)(\mathbf{c} + d) = ac + \dots$

Outside two terms: $(\mathbf{a} + b)(c + \mathbf{d}) = ac + ad + \dots$

Inside two terms: $(a + \mathbf{b})(\mathbf{c} + d) = ac + ad + bc + \dots$

Last two terms: $(a + \mathbf{b})(c + \mathbf{d}) = \boxed{ac + ad + bc + bd}$

Example: $(x + 2)(x - 7) = x^2 - 7x + 2x - 14 = \boxed{x^2 - 5x - 14}$

2. **The Quadratic Formula** \Rightarrow Used for solving *quadratic equations*, that is, equations of the form: $ax^2 + bx + c = 0$

*note: the equation must be in this form (all terms on one side, zero on the other) to use the quadratic formula!

Given the quadratic equation: $ax^2 + bx + c = 0$, the solution(s) is/are:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

We typically write this as: $x = \boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Two (if $b^2 - 4ac > 0$), one (if $b^2 - 4ac = 0$) or no (if $b^2 - 4ac < 0$) solutions are possible

Example: Solve $3x^2 - 2x + 1 = 7$ for x

First, get all terms on one side of the equation: $3x^2 - 2x - 6 = 0$, now we use the Q.F.

where $a = 3$, $b = -2$ and $c = -6$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)} \quad \dots \text{don't forget PEMDAS!!!} \\ &= \frac{2 \pm \sqrt{4 + 72}}{6} = \frac{2 \pm \sqrt{76}}{6} = \frac{2 \pm 2\sqrt{19}}{6} = \frac{\cancel{2}(1 \pm \sqrt{19})}{\cancel{6}3} = \frac{1 \pm \sqrt{19}}{3}, \text{ so the two} \end{aligned}$$

solutions are... $x = \boxed{\frac{1 + \sqrt{19}}{3} \approx 1.786}$ and $x = \boxed{\frac{1 - \sqrt{19}}{3} \approx -1.120}$

3. **Factoring** \Rightarrow Used to solve equations (including quadratic equations)

(a) In general, factoring is like dividing

Example: Factor $6x^5 - 10x^3 + 4x^2 - 16x$

\rightarrow Look at each term in the expression and find the greatest common divisor (gcd):

For our expression it's $\boxed{2x}$

\rightarrow Next, *divide* each term in the expression by the gcd and write the gcd outside the parentheses: $\boxed{2x(3x^4 - 5x^2 + 2x - 8)}$

(b) In particular, factoring quadratics involves trial and error.

Example: Factor $x^2 + x - 12$, it will look like $(x \pm \underline{\quad})(x \pm \underline{\quad})$

\rightarrow Find two numbers that multiply to -12

\rightarrow The possibilities are: 1 & -12, -1 & 12, 2 & -6, -2 & 6, 3 & -4 or -3 & 4

\rightarrow We also need the two numbers to add up to 1 (the coefficient of the x term), so the two numbers are -3 & 4.

So $x^2 + x - 12 = \boxed{(x - 3)(x + 4)}$

As you can see, this can be tedious, but it will come more quickly with practice. Sometimes you won't be able to come up with the two numbers needed, and the QF will be quicker!

4. **Solving Equations** \Rightarrow There are many ways of solving equations because there are many different types of equations! Here are some key ideas to keep in mind...

(a) Liberate the variable (we'll call her x)! To do this you must undo what ever is being done to x by using the inverse operations.

◇ Addition & Subtraction

◇ Multiplication & Division

◇ Exponents & Radicals (Roots)

(b) Be fair! Whatever you do to one side of the equation, do to the other side as well.

(c) Share the wealth! If you multiply (or divide) each side by a number, multiply (or divide) *each term* by that number.

(d) There is strength in numbers! If there is more than one x in the equation, get all of the x 's to one side of the equation.

(e) Do nothing! Multiplying by one or adding zero are often important tricks to have up your sleeve!