

Algebra Refresher Workshop: Key Ideas

Most algebraic mistakes boil down to a misunderstanding of one of the following:

Key Idea #1: *The Order of Operations*

One way to remember the order \Rightarrow “**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally” (**P.E.M.D.A.S.**)

Parentheses (and other grouping symbols)

Exponents

Multiplication & **D**ivision (from left to right)

Addition & **S**ubtraction (from left to right)

Examples:

1. $-3^4 = -3 \cdot 3 \cdot 3 \cdot 3 = \boxed{-81}$ \rightarrow do exponent, then negate
2. $(-3)^4 = (-3)(-3)(-3)(-3) = \boxed{81}$ \rightarrow negative inside parentheses, so negate the 3, then raise it to 4th power)
3. $2 - 3 \cdot 4 \div 2 + 7 = 2 - 12 \div 2 + 7 = 2 - 6 + 7 = -4 + 7 = \boxed{3}$
 \rightarrow multiply, divide, subtract, add
4. $12 \div (5 - 2)^2 - (2^2 - 5) \div 3 \rightarrow$ first do what’s in the parentheses, note 2nd parenth. has an exponent in it!
 $= 12 \div (3)^2 - (4 - 5) \div 3 = 12 \div (3)^2 - (-1) \div 3 \rightarrow$ next do remaining exponent
 $= 12 \div 9 - (-1) \div 3 \rightarrow$ next do two divisions
 $= \frac{4}{3} - \left(-\frac{1}{3}\right) = \boxed{\frac{5}{3}}$
5. $\sqrt{5^2 - 4^2} + \sqrt{3^2 + 2^2} \rightarrow$ square root is a grouping symbol like parentheses so start with what’s inside. Inside the square roots there are exponents, so do those first
 $= \sqrt{25 - 16} + \sqrt{9 + 4} \rightarrow$ now complete the insides of the square roots
 $= \sqrt{9} + \sqrt{13} = \boxed{3 + \sqrt{13}}$ \rightarrow 2nd square root can’t be simplified
6. $\frac{7 - 4}{3 \cdot 2 + 4^2} \rightarrow$ a fraction is a grouping symbol, the numerator (top) is grouped together and the denominator (bottom) is in a separate “group”. I’ll first simplify the numerator
 $= \frac{3}{3 \cdot 2 + 4^2} \rightarrow$ you may be tempted to cancel the 3’s, but we can’t because there is not a *factor* of 3 in the denominator (we’ll talk more about this in a moment). Next do exponent
 $= \frac{3}{3 \cdot 2 + 16} \rightarrow$ now multiplication, then addition, $= \frac{3}{6 + 16} = \boxed{\frac{3}{22}}$

Key Idea #2: Commonly Used Terminology

Expression vs. Equation , Term vs. Factor and Constant vs. Variable

An **EXPRESSION** is made up of variables, numbers, grouping symbols, operation signs (+, −, ×, ÷) and exponents, while an **EQUATION** is built from expressions using one or more equal sign.

Examples:

1. $3x^2 - 2x + 1$ is an *expression*, while $3x^2 - 2x + 1 = 7$ is an *equation*. You could solve the equation for x (figure out what value of x would make it true), but we can't solve an expression.
2. $\frac{4ab^3 - a}{2a^2 - 5b} - \sqrt{16b^3 + 5}$ is an *expression*, while $\frac{4ab^3 - a}{2a^2 - 5b} = \sqrt{16b^3 + 5}$ is an *equation*.

TERMS are the parts of an expression separated by + or − signs, while **FACTORS** are parts of an expression being multiplied or divided.

Examples: Find the *terms* and *factors* of the following expressions

1. $3x^2 - 6x - 15$
The expression has 3 terms: **$3x^2$** , **$-6x$** , and **-15** (we could also have said **$6x$** and **15**).
The expression has 2 factors, but to see them we must first factor it:
 $3x^2 - 6x - 15 = 3(x^2 - 2x - 5)$, so the two factors are **3** and **$x^2 - 2x - 5$** .
2. $3a^2b$
The expression has 1 term, **$3a^2b$**
The expression has 4 factors, **3** , two factors of **a** and one factor of **b** .

CONSTANTS are values that are fixed. Numbers are constants, but letters are also often used to represent constants. **VARIABLES** are values that can change. They are represented using letters.

Examples:

1. Constants: 3, $\frac{7}{2}$, -12 , $\sqrt{7}$, π , c (the speed of light)...
2. Variables: we often use x and y , but the velocity of a car, v or the number of products produced, n could also be variables!

Why is this terminology important?

→ There is an important distinction between combining or canceling like *terms* and like *factors*. Confusing these often leads to common mistakes.

Example: For #6 on the previous page I said we couldn't cancel the 3 because there wasn't a *factor* of 3 in the denominator. One of the terms in the denominator had a *factor* of 3, but this is not enough. In order to cancel the 3, we need a *factor* of 3 in each and every term in both the numerator and the denominator!

Key Idea #3: *Inverse Operations*

These are operations that "undo" each other. It is very important to know how to use these properly when solving equations!

- ◇ Addition & Subtraction
- ◇ Multiplication & Division
- ◇ Exponents & Radicals (Roots)

Examples:

1. Eliminate the 5: $5x = 75$

Since there is a 5 is being *multiplied* by x to undo this we will *divide* both sides by 5

$$\frac{5x}{5} = \frac{75}{5} \Rightarrow x = 15$$

2. Eliminate the radical: $\sqrt[3]{x} = 4$

To eliminate the *radical* we will use an *exponent*. To eliminate a cubed (3rd) root, we raise both sides to the 3rd power.

$$(\sqrt[3]{x})^3 = (4)^3 \Rightarrow x = 64$$

3. Eliminate the 17: $x + 17 = -12$

Since 17 is being *added* to x , to eliminate it, we will *subtract* 17 from each side.

$$\begin{array}{r} x + 17 = -12 \Rightarrow x = -29 \\ -17 \quad -17 \end{array}$$

Keeping these three Key Ideas in mind...

Common Algebra Mistakes

1. $(x + y)^2 = x^2 + y^2$ → These differ because of the order of operations:

The expression on the left adds x to y , then squares the result

The expression on the right squares both x and y , then adds the result.

2. $(-x)^4 = -x^4$ → Also the order of operations:

The left side negates x , then raises that answer to the 4th power

The right side raises x to the 4th power, then negates the result

3. $\sqrt{a^2 + b^2} = a + b$ → you tell me what's wrong!

4. $\frac{a + b}{a} = \frac{1 + b}{1}$ → there is no *factor* of a on the top, so we can't cancel the a 's.

What we could do is to split this into two fractions: $\frac{a}{a} + \frac{b}{a} = 1 + \frac{b}{a}$

5. $\frac{a}{a + b} = \frac{1}{1 + b}$ → you tell me what's wrong!

6. To solve the equation $7x = 10$, subtract 7 from each side to get: $x = 3$

→ what's wrong with this?

⇒ Come up with numerical examples to show that each of the above is FALSE!

What other algebra questions do you have?

Need more practice? Come to the QSC! You can sit down to work with a tutor or check out an Algebra textbook to practice on your own.

We are located in UW2-134