Managing Project Portfolios: Statistical vs Managerial Spillover*

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Abstract

Choosing projects to undertake is a general managerial problem that ranges from determining which units or divisions to establish within a firm, or which acquisitions or alliances to pursue, to lower-level functions such as which R&D, financial ventures, or marketing campaigns to greenlight.

Our paper analyzes the problem of selecting a portfolio given a pool of projects under a research budget, allowing for value spillover across projects. We distinguish between two types of spillover: *managerial spillover*, due to the presence of common managerial resources or real assets, and *statistical spillover*, when news about the profitability of a project is informative about other projects. This distinction, largely overlooked in the literature, has tangible implications for management. Statistical spillover is consistent with decentralized, case-by-case project undertaking *as long as information flows freely across divisions*. Managerial spillover, however, requires that projects be undertaken within the same division and *assessed in blocks*: The combined savings from passing on two projects at once may outweigh their marginal contributions.

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1 Introduction

Choosing a portfolio of projects is a central managerial problem. In corporate strategy, managers must decide which business units or divisions to establish within their firms, and which alliances or acquisitions to pursue. At a more disaggregate level, division managers must decide which R&D projects or marketing campaigns to greenlight, and venture capitalists must decide which businesses to support.

Our paper analyzes the problem of selecting a portfolio from a pool of projects under a research budget and allowing for value spillover across projects. Projects must be selected at the *interim* stage, where the manager possesses some preliminary information about their profitability (preliminary reports, expert assessments, peer reviews, etc.) but before their value is fully realized. This timing allows us to make a distinction between two types of spillover that has been largely overlooked in the literature: *managerial spillover*, whereby the appreciation of a project leads to the appreciation of other projects under the same management structure due to common managerial resources or real assets; and informational or *statistical spillover*, whereby preliminary reports about the profitability of a project convey information about the profitability of other projects.

Some examples may help shed light on this distinction:

• A farmer is considering whether to buy one or two neighboring plots of land. If one of the plots is sufficiently large to make it profitable to invest in large, powerful farm equipment, the same equipment can be employed to extract more value out of the neighboring plot; this is a managerial spillover. At the same time, analyzing the chemical composition of the soil in one plot to predict its fertility will shed information on the fertility of the neighboring plot, as proximity implies similar soil properties; this is a statistical spillover.

• A drilling company is interested in bidding for one or two neighboring oil tracts. Proximity of the tracts can help the company mobilize resources across oil platforms (managerial spillover) and makes findings on the properties of the soil in one tract informative about the likelihood of finding oil in the neighboring tract (statistical spillover).

• An investor is looking to fund two development projects in the same area. Proximity allows the investor to mobilize construction equipment and labor across projects (managerial spillover), while news about the real estate market gathered from analysis on one project is useful to assess the other one (statistical spillover).

• A pharmaceutical company is evaluating research on two alternative treatments for a disease. Treatment A is based on the hypothesis that the disease is bacterial, while treatment B, that it is viral. Lab equipment and staff can be shared across the two research teams (managerial spillover). A successful trial from treatment A is good news about it but bad news about treatment B, and viceversa (statistical spillover).

While this understudied distinction might seem subtle, it has tangible managerial implications. We show that, under statistical spillover, projects can be assessed on a decentralized, case-by-case basis, and undertaken autonomously (given budgetary approval) *as long as information flows freely across divisions*. If relevant preliminary findings are shared across all units, each unit can assess and carry out their own project (if approved). Capturing managerial spillover, however, requires that the projects be undertaken within the same management unit and be *assessed in blocks*: The combined savings from passing on two projects at once may outweigh their marginal contribution; individual assessments can be misguiding.

While an important portion of the management literature overlooks managerial spillover (Arora and Gambardella, 1994; Adner and Levinthal, 2004; Trigeorgis and Reuer, 2017), there is a rich line of work that accounts for such spillover. Fox et al. (1984) analyzes projects whose present value have a non-linear impact on profit; however, profit impacts are *deterministic*. Moreover, they exclude statistical spillover, as successes or failures are independent across projects. Ghasemzadeh et al. (1999) specifies a linear profit and accommodates value spillover as precursor/successor constraints on project choice. Dickinson et al. (2001) represents project dependency by means of a (not necessarily symmetric) square matrix; the value of a portfolio and projects' interactions are additive and *deterministic*. Liesio et al. (2008) represents project dependency by introducing dummy projects with value and cost given by the value and cost interaction across projects. While this approach can accommodate specific spillovers across a small number of projects, the number of dummy projects needed to account for the more global managerial spillover grows exponentially with number of projects under consideration.

Statistical spillover has received far less attention in the literature and is largely understudied. Arora and Gambardella (1994) stresses the importance for industrial research of generalized and abstract knowledge, the type of knowledge with multiple applications. Loch and Kavadias (2002) studies a multi-period, multi-product firm under uncertainty about the market conditions for their different products. They allow for correlation across market conditions (Proposition 4), but their analysis is carried out from the point of view of time 0, the *ex-ante* stage. Thus, all expectations are *unconditional*. To assess statistical spillover, managerial decisions must be made at the interim stage, conditional on preliminary or noisy information.

The paper is organized as follows. The next section presents the general problem. To shed more light on the managerial implications of the two types of spillover, we isolate statistical spillover in section 3 and managerial spillover in section 4. Section 5 extends the analysis in section 4 to accommodate asymmetric value distributions. Section 6 concludes.

2 The General Problem

There is a pool of n projects, denoted by $N = \{1, \ldots, n\}$. The individual present value (PV) of undertaking project i is denoted by $v_i \in [\underline{v}_i, \overline{v}_i]$, where $\overline{v}_i > \underline{v}_i \geq 0$; project i's cost is $c_i > 0$. Given a profile of project values $v = (v_1, \ldots, v_n)$, if project portfolio $A \subseteq N$ is chosen, the ex-post profit for the general manager is given by:

$$\Pi(A, v) := \sum_{i \in A} \left[v_i \left(1 + \theta \sum_{j \in A \setminus \{i\}} v_j \right) - c_i \right], \tag{1}$$

where $\theta > 0$ is the degree of *managerial spillover* (MS) between projects. Handling projects in house, with shared managerial resources, adds an interaction effect to the projects' value; θ captures the degree of this interaction.

The specification in (1) is inspired by the model of knowledge accumulation of Cohen and Levinthal (1989) and the synergy specifications of Fox et al. (1984) and Loch and Kavadias (2002). Projects spill their PV over to other projects' PV. This effect is proportional, so spillover can increase the revenue from a successful project but cannot cause an unsuccessful project to succeed: If a project always yields a gross value of 0, no amount of spillover from other projects will change that.

There's uncertainty about projects' PV's. Unless otherwise stated, we allow for PV's to be correlated across projects; this is the source of *statistical spillover* (SS). Two neighboring farms will have similar yields based on similarities in their soil properties; the state of the real estate market will affect two neighboring development projects.

Portfolios must be chosen before PV's are realized, on the basis of preliminary information from experience with similar projects, research trials, peer reviews or reports, etc., and speak to how promising a project is. We denote project i's signal by

$s_i \in [\underline{s}_i, \bar{s}_i]$, where $\bar{s}_i > \underline{s}_i \geq 0$. Given v_i, s_i is drawn from the conditional distribution with pdf $f_i(s_i|v_i)$.[1]

Employing Bayes' rule, we compute the posterior distribution of the value of the projects given the signals. Since we allow for correlation of PV's, the signals are generally not independent under their unconditional distribution. For each i, given a profile of signals $s = (s_1, \ldots, s_n)$, define the function $\phi_i(s)$ that gives the expected value of project i given the signals: $\phi_i(s) = E[v_i|s]$. Our SS is the dependence of $\phi_i(s)$ on s_{-i}, where s_{-i} is the profile of signals for projects other than i. If two projects are positively correlated and we get a high signal from one but a low signal from the other, the latter's low signal puts a damper on the optimism from the former's high signal (and viceversa). A low productivity outcome from a farm can be attributed to a bad weather draw or contained pest (and thus discounted) in light of a high productivity outcome from a neighboring farm with similar soil and technology. On the other hand, if the projects are negatively correlated, the low signal from the second project is even better news about the first one. In our pharmaceutical example, a failed trial for treatment A is good news for treatment B.

Thus, a project's signal can be relevant about other projects' value *even in the absence of MS*. Of course, under independence, $\phi_i(s)$ is a function only of s_i: $\phi_i(s) = \phi_i(s_i)$.

Loch and Kavadias (2002) allows for MS to be project-pair specific. In our setting, this parameter is not project specific but "manager specific." We can account for project-pair specific synergies through correlation between PV's. However, project-pair specific MS parameters cannot capture our SS.[2]

We assume that, for each i, the signals (v_i, s_i) are *affiliated*. This means that higher project values are more likely given higher signals. In other words, a high signal is good news about the value of the corresponding project. We assume that it is in fact strictly good news, in the sense that $\phi_i(s_i, s_{-i})$ is strictly increasing in s_i for every i.

Assumption (Information structure)**.** PV's $v_1, \ldots, v_n$ are random variables; signals $s_1, \ldots, s_n$ are continuous random variables, each s_i drawn conditional on v_i; for each i, $\phi_i(s_i, s_{-i})$ is strictly increasing with respect to s_i.

[1]The assumption that the distributions of signals are continuous is made for simplicity of the exposition; continuous distributions allow us to ignore the possibility of ties. Discrete distributions can be accommodated in the analysis.

[2]As indicated in (2) below, the MS term between projects i and j is $\theta E[v_i v_j|s]$. If we define $\theta_{ij}(s) := \theta\phi_i(s)\phi_j(s)E[v_i v_j|s]^{-1}$, this term becomes $\theta_{ij}(s)\phi_i(s)\phi_j(s)$. However, the SS remains insofar as $\phi_i(s)$ depends on s_j, and viceversa.

Let $B > 0$ be the manager's *research budget*. Given a profile of signals $s = (s_1, \ldots, s_n)$, the manager chooses a project portfolio $A \subseteq N$ in order to maximize interim-expected profit (given the projects' signals):

$$\pi(A, s) := E\left[\Pi(A, v)|s\right] = \sum_{i \in A} [\phi_i(s) - c_i] + \theta \sum_{i \in A} \sum_{j \in A \setminus \{i\}} E[v_i v_j | s] \qquad (2)$$

subject to the constraint $\sum_{i \in A} c_i \leq B$.

Example 1. A manager has $n = 4$ projects for consideration. Each project i can either be successful, in which case $v_i = 1$, or a dud, in which case $v_i = 0$. The projects are independent, and their signals $s_1, \ldots s_4 \in [0, 1]$ are their probability of success; here, $\phi_i(s_i) = s_i$. Given costs and θ, the profit from selecting portfolio $A = \{2, 4\}$ is:

$$\pi(A, s) = s_2 - c_2 + s_4 - c_4 + 2\theta s_2 s_4.$$

Example 2. Assume now that the projects' values are independently and uniformly distributed on the interval $[0, 1]$, and that, given v_i, signal s_i is distributed uniformly on the interval $[0, v_i]$. Then, $\phi_i(s_i) = \frac{s_i - 1}{\ln(s_i)}$. The profit from selecting portfolio $A = \{2, 4\}$ is now:

$$\pi(A, s) = \frac{s_2 - 1}{\ln(s_2)} - c_2 + \frac{s_4 - 1}{\ln(s_4)} - c_4 + 2\theta \frac{s_2 - 1}{\ln(s_2)} \frac{s_4 - 1}{\ln(s_4)}.$$

Example 3. There are two projects. With probability $\frac{1}{2}$, v_1, v_2 are drawn from a uniform distribution on $[0, 1]$, and with probability $\frac{1}{2}$, from a uniform distribution on $[0, 2]$. PV's above 1 can only come from the second distribution; but values below 1 are consistent with both distributions. Given v_i, signal s_i is uniformly drawn from $[0, v_i]$. Imagine that $1 \geq s_1 > s_2$; then,

$$\phi_2(s_1, s_2) = \frac{-5\ln(s_1)(1 - s_2) + \ln(2)}{5\ln(s_1)\ln(s_2) - \ln(2)\ln(s_1) - \ln(2)\ln(s_2) + \ln(2)^2}.$$

If, instead, $s_1 > 1 > s_2$, we get:

$$\phi_2(s_1, s_2) = \frac{2 - s_2}{\ln(2) - \ln(s_2)} = \phi_2(s_2).$$

For instance, if $s_2 = 0.4$ and $s_1 = 0.9$, we get $\phi_2(s_1, s_2) = 0.6039$; but if $s_1 = 1.4 > 1$, we know that both PV's are drawn independently from the uniform distribution on

$[0,2]$, that the low s_2 is simply a "bad draw," and so $\phi_2(s_1,s_2) = 0.9941$ — even if the signal s_2 is the same, the higher s_1 is good news about both PV's and puts the lower s_2 into perspective.

There are two effects that our framework omits. (1) As our analysis is static in nature, we do not allow managers to learn from past "mistakes" when a project's realized PV is lower than expected. (2) Low-profit projects may be valuable if they provide know-how for managers to help other projects succeed. Nonetheless, the latter can be captured in essence as negative correlation across projects.

The next lemma establishes that the general manager's problem is a well-behaved programming problem.

Lemma 1. *The increment in profit from adding a project is larger the larger is the underlying portfolio. Formally, for each signal profile s,* $\pi(A,s)$ *is supermodular in* A*: For any two* $A \subseteq B \subseteq N$ *and any* $j \notin A$, $\pi(B \cup \{j\}, s) - \pi(B,s) \geq \pi(A \cup \{j\}, s) - \pi(A,s)$.

However, finding the optimal portfolio can be cumbersome. We can form 2^n portfolios from a pool of n projects; with only 10 projects on the table, we have 1,024 portfolios to assess. In fact, the manager's problem of maximizing interim-expected profit subject to the budget constraint is NP hard. Lemma 1 allows this problem to be solved as a size-constrained supermodular maximization problem; see, for instance, Nagano et al. (2011).[3]

Proposition 1. *Consider the set of possible sizes of affordable portfolios,* $N_B := \{|A| : A \subseteq N, \sum_{i \in A} c_i \leq B\}$. *For each* $k \in N_B$*, given a profile of signals s, consider the problem of maximizing* $\pi(A,s)$ *subject to the constraint* $|A| = k$*, and let* $A_k^*(s)$ *be a solution to this problem. Then, a solution to the manager's problem,* $A^*(s)$*, is given by the most profitable* $A_k^*(s)$ *across* $k \in N_B$: $A^*(s) = \arg\max\{\pi(A_k^*(s), s) : k \in N_B\}$.

In what follows, we isolate the impact of SS and MS in turn to shed further light on their managerial implications for portfolio selection.

3 Isolating Statistical Spillover

We start by considering the benchmark with no MS, namely the case with $\theta = 0$. Here, expected profit is additively separable across projects: For each $A \subseteq N$ and

[3] Nagano et al. (2011) analyzes the equivalent problem of minimizing submodular functions.

signal profile s,

$$\pi(A,s) = \sum_{i\in A} [\phi_i(s) - c_i].$$

As previously noted, we can have SS without MS. If two projects are correlated, their signals are informative of both of them. While expected profit reduces to the sum of expected net PV's, each of these expectations is *conditional (in principle) on the entire profile of signals*.

The following notation will be useful. Given a profile of signals s, let i_1 be the index of the project with the highest expected net PV: $\phi_{i_1}(s) - c_{i_1} = \max\{\phi_i(s) - c_i : i = 1, \ldots, n\}$; similarly, let i_k for $k = 2, \ldots, n$ be the index of the project with the k-th highest expected net PV.[4]

The next proposition characterizes the optimal portfolio in the SS benchmark: We want to undertake the most-promising projects we can afford, *assessing each project's net PV based on the signals of all other (relevant) projects*.

Proposition 2. *Assume that $\theta = 0$, so that there are no managerial spillovers. Given a profile of signals s, define $k(s,B) = \max\{k = 1, \ldots, n : \phi_{i_k}(s) > c_{i_k}$ and $\sum_{j=1}^{k} c_{i_j} \leq B\}$. Then, $A^0(s) := \left\{i_1, \ldots, i_{k(s,B)}\right\}$ is the optimal project portfolio.*

Example 1 (Continued)**.** Project i's net PV is $s_i - c_i$, its probability of success net of its cost. Assume that $c_1 = \cdots = c_4 = 1$, $B = 2$, and $s_1 > s_2 > s_3 > 1 > s_4$. Then, $k(s,B) = 2$, and $A^0(s) := \{1,2\}$. With $B = 3$ we get $k(s,B) = 3$, and $A^0(s) := \{1,2,3\}$; however, the answer will not change if we extend the budget to $B = 4$, as the last project has a negative expected net PV.

Example 2 (Continued)**.** Here, project i's net PV is $\frac{s_i - 1}{\ln(s_i)} - c_i$. Assume that $c_1 = \cdots = c_4 = 1$, $B = 2$, and $\frac{s_1-1}{\ln(s_1)} > \frac{s_2-1}{\ln(s_2)} > \frac{s_3-1}{\ln(s_3)} > 1 > \frac{s_4-1}{\ln(s_4)}$. Again, $k(s,B) = 2$ and $A^0(s) := \{1,2\}$.

Example 3 (Continued)**.** Project 1's net PV, if $s_2 < s_1 < 1$, is:

$$\frac{-5\ln(s_2)(1-s_1) + \ln(2)}{5\ln(s_1)\ln(s_2) - \ln(2)\ln(s_1) - \ln(2)\ln(s_2) + \ln(2)^2} - c_1;$$

[4] To be utterly precise, we should write $i_1(s), \ldots, i_n(s)$, as this ranking is contingent on the profile of signals. We suppress this dependence from the notation in the interest of simplicity.

otherwise, if $s_1 > 1$, we have:

$$\frac{2 - s_1}{\ln(2) - \ln(s_1)} - c_1.$$

Similarly for s_2.

In the absence of spillover of any kind, the optimal portfolio consists of the highest-ranked projects with positive net PV that we can afford. Projects can be assessed separately, so general managers can delegate that task to the corresponding unit. These units can also carry out the project in complete autonomy subject to budgetary approval.

Statistical spillover is consistent with such decentralized project assessment and undertaking *as long as information flows freely across units*; the general manager only needs to make sure that divisions share all the relevant signals so that they can compute expected PV's accurately.

For $\theta > 0$, interim-expected profit surpasses the sum of expected net PV's: For every $A \subseteq N$ and s, $\pi(A, s) \geq \sum_{i \in A}[\phi_i(s_i) - c_i]$. If the budget constraint is not binding, or if projects' costs are equal and their PV's are independent, we can show that $A^0(s) \subseteq A^*(s)$: All projects that can "stand on their own" remain profitable in the presence of MS. Intuitively, such spillover can only increase said projects' value. The question is which *other* projects to add to the portfolio, projects that do not stand on their own but become profitable given the spillover from other projects.

However, with a binding budget and correlated projects, it may pay to drop a project that "stands on its own" to make room in the budget for a project with lower expected net PV but with higher MS.

Example 4. There are 3 projects, which can be either successful or fail. The signals are their value in the event of success, and successes and failures are equally likely: $v_i|s_i = s_i$ with probability $\frac{1}{2}$ and $v_i|s_i = 0$ with probability $\frac{1}{2}$. However, the PV of projects 1 and 2 are negatively correlated, while 1 and 3 are positively correlated; see Table 1. Signals are *i.i.d.* uniform on $[0, 1]$; costs are $c_1 = c_2 = c_3 = 1$, and $B = 2$. With $s_1 > s_2 > s_3$, then $A^0(s) = \{1, 2\}$. However, $E(v_1 v_2 | s_1, s_2) = 0$ and $E(v_1 v_3 | s_1, s_3) = \frac{s_1 s_3}{2}$; as long as s_3 is not too low — more precisely, for $s_3 > \frac{s_2}{1+\theta s_1}$ —, we have that $A^*(s) = \{1, 3\}$.

$v_1 \setminus v_2, v_3$	$v_2 = 0$	$v_2 = s_2$	$v_3 = 0$	$v_3 = s_3$
$v_1 = 0$	0	1/2	1/2	0
$v_1 = s_1$	1/2	0	0	1/2

Table 1: Joint distribution of v_1, v_2, v_3 in Example 4.

4 Managerial Spillover and Independent, Symmetric Projects

To highlight the impact of MS, we assume that projects are independent (so that there are no SS). We begin with the case of ex-ante identical or symmetric projects in the sense that their values are *i.i.d* and their costs are equal. Thus, we can drop the subindex i from both ϕ_i and c_i, and we can rank projects by simply ranking their signals.

The next lemma establishes that we can always improve upon a portfolio that has a lower-ranked project and excludes a higher-ranked project by simply swapping the two projects. In other words, we can always improve upon a portfolio by replacing any of its projects with an excluded better one.

Lemma 2. *Fix a profile of signals s and relabel the projects if necessary so that $s_1 > s_2 > \cdots > s_n$. For every project i in portfolio A and every project j excluded from A, if j is ranked higher than i (so that $i > j$), then replacing i with j increases the profit from the portfolio: $\pi((A \setminus \{i\}) \cup \{j\}, s) > \pi(A, s)$.*

This lemma implies that we can characterize the optimal portfolio by means of a cutoff: The optimal portfolio is of the form $\{1, \ldots, j\}$ for some project $j \in N$ (unless it is empty). Thus, we can reduce the manager's present problem to the following maximization problem:

$$\max_{j \in \{1,\ldots,n\}} \pi(\{1, \ldots, j\}, s)$$

subject to the constraint $jc \leq B$. We assume that $\lfloor \frac{B}{c} \rfloor \leq n$, where $\lfloor \frac{B}{c} \rfloor$ is the integer part of $\frac{B}{c}$ — the number of projects that the firm can afford.

Example 1 (Continued)**.** Imagine that the signals are $s_1 = 0.9$, $s_2 = 0.6$, $s_3 = 0.4$, and $s_4 = 0.1$. Costs are $c_1 = c_2 = c_3 = c_4 = 0.35$, and $\theta = 0.5$. Figure 1 depicts the profit from the portfolios of the form $\{1, \ldots, i\}$ for $i = 1, \ldots 4$. If the budget constraint is not binding, the optimal portfolio is the portfolio $A^*(s) = \{1, 2, 3\}$.

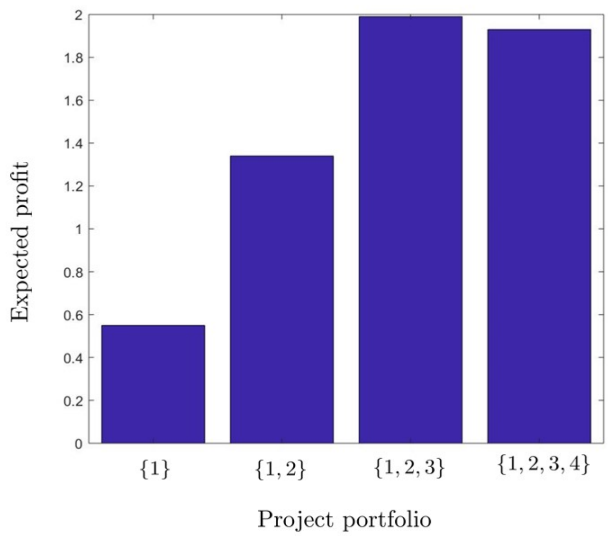

Figure 1: Expected profit for the relevant portfolios given the signals in Example 1.

Example 2 (Continued)**.** Let costs, θ, and signals be the same as in Example 1 above — but under the distributions of Example 2. Now, if the budget constraint is slack, the optimal portfolio is the full portfolio, $A^*(s) = \{1, 2, 3, 4\}$; see Figure 2.

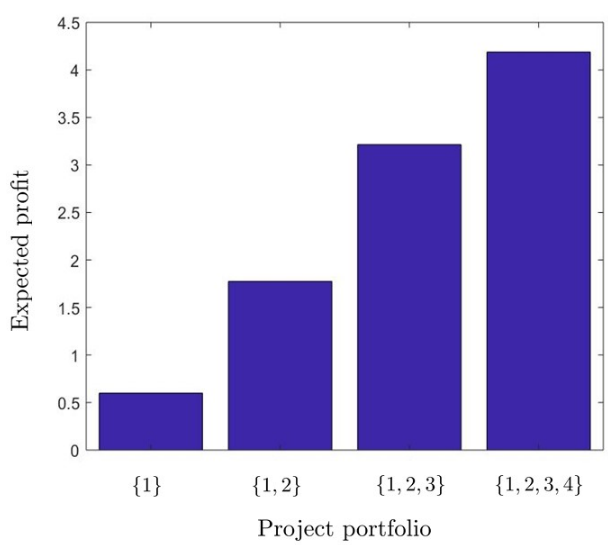

Figure 2: Expected profit for the relevant portfolios given the signals in Example 2.

In both examples 1 and 2, the graphical representation of the profit from the portfolios of the form $\{1, \ldots, i\}$ is *single-peaked*. This makes finding the optimal portfolio particularly easy: Starting from the lowest-ranked project, discard projects one at a time while doing so raises profit; as soon as profit would start going down, stop if you are within your budget and keep going until you meet your budget.

Unfortunately, this is not a general property of the problem, but rather a special feature of these examples. The following example shows that, under MS, interim-expected profit may not be single-peaked.

Example 5. In the same environment as in Example 1, but with only $n = 3$ projects, imagine that the signals are $s_1 = 0.99$, $s_2 = 0.3$, $s_3 = 0.29$; costs are $c_1 = c_2 = c_3 = 0.64$, and $\theta = 0.5$. Figure 3 depicts the profit from the portfolios of the form $\{1, \ldots, i\}$ for $i = 1, \ldots 3$. The full portfolio is more profitable than the portfolio $\{1, 2\}$; however, the best portfolio to undertake is project 1 alone.

Thus, accounting for MS requires *evaluating projects in blocks*, and introduces the following asymmetry in the selection process. We start with the lowest-ranked project, where the managerial spillover is maximal. If including the lowest-ranked project decreases the profit from the portfolio of all the other projects, it is hopeless:

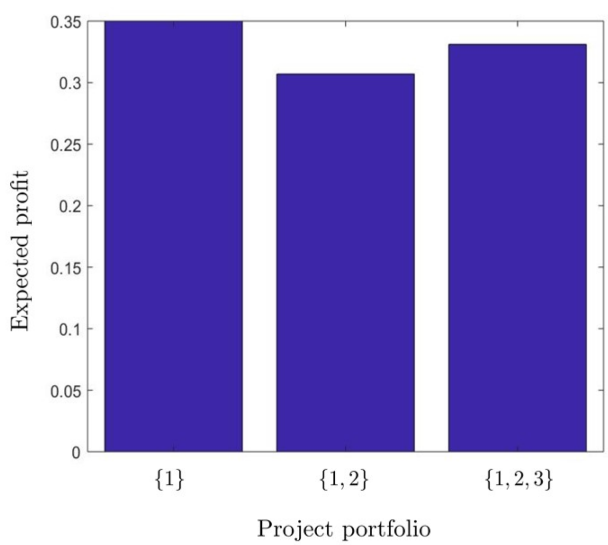

Figure 3: Expected profit for the relevant portfolios given the signals in Example 5.

Even with the largest possible spillover, its contribution to revenue falls short of its cost. Then, it can be safely discarded. However, said contribution being positive *does not mean that the project should be automatically adopted*: It may still be profitable to discard it together with other projects, *even if we already are within our budget*. The intuition is that, while spillover may make it more profitable to undertake such a project than to abandon it *if all superior ones are undertaken*, the savings in aggregate cost from abandoning multiple projects *at once* may outweigh the corresponding loss in revenue.

The next proposition provides an algorithm to construct the optimal portfolio given a profile of signals. For this algorithm, we will employ the following measure of incremental profit.

Definition 1 (Block incremental profit)**.** Given a portfolio A, a project $i \in A$ such that $i > 1$, and a profile of signals s, define the *block incremental profit (BIP) of dropping i from A, $\nu(i, A, s)$*, as the change in expected profit from discarding *i together with all projects in A ranked below i, if any*: $\nu(i, A, s) := \pi(\{1, \ldots, i-1\}, s) - \pi(A, s)$.

Ranking the projects by signal from highest to lowest, Lemma 2 allows us to focus on the highest-ranked projects that we can afford.

Proposition 3. *Fix a profile of signals s and relabel the projects if necessary so that $s_1 > s_2 > \cdots > s_n$. The following algorithm constructs $A^*(s)$.*

1. *Step 1: Compute $\nu(\lfloor \frac{B}{c} \rfloor, \{1, \ldots, \lfloor \frac{B}{c} \rfloor\}, s)$, the BIP of dropping the lowest-ranked project that we can afford.*
   (a) *If $\nu(\lfloor \frac{B}{c} \rfloor, \{1, \ldots, \lfloor \frac{B}{c} \rfloor\}, s) > 0$, set $A_1 := \{1, \ldots, n-1\}$ and move to step 2.*
   (b) *If $\nu(\lfloor \frac{B}{c} \rfloor, \{1, \ldots, \lfloor \frac{B}{c} \rfloor\}, s) < 0$, set $A_1 := N$ and move to step 2.*
2. *Step $k = 2, \ldots, \lfloor \frac{B}{c} \rfloor - 1$: Compute $\nu(\lfloor \frac{B}{c} \rfloor - k + 1, A_{k-1}, s)$, the BIP of dropping project $n - k + 1$ from the portfolio of remaining projects under consideration.*
   (a) *If $\nu(\lfloor \frac{B}{c} \rfloor - k + 1, A_{k-1}, s) > 0$, set $A_k := \{1, \ldots, \lfloor \frac{B}{c} \rfloor - k\}$ and move to step $k + 1$.*
   (b) *If $\nu(\lfloor \frac{B}{c} \rfloor - k + 1, A_{k-1}, s) < 0$, set $A_k := A_{k-1}$ and move to step $k + 1$.*
3. *Step $\lfloor \frac{B}{c} \rfloor$: Compute $\pi\left(A_{\lfloor \frac{B}{c} \rfloor - 1}, s\right)$*

(a) *If* $\pi\left(A_{\lfloor \frac{B}{c} \rfloor -1}, s\right) < 0$, *set* $A^*(s) := \emptyset$ *and stop.*

(b) *If* $\pi\left(A_{\lfloor \frac{B}{c} \rfloor -1}, s\right) > 0$, *set* $A^*(s) := A_{\lfloor \frac{B}{c} \rfloor -1}$ *and stop.*

The algorithm reflects the asymmetry in the decision process. If a project yields a negative incremental profit to the portfolio of superior projects, then said project can be safely discarded — sub-step (a). However, a project that yields positive incremental profit to said portfolio is not automatically adopted, since it may be profitable to discard it along with other projects at once — sub-step (b).

The asymmetry in the decision rule disappears if the profit function is single peaked, as in examples 1 and 2. In this case, we can simplify our search process further. In order to do this, we introduce the following definitions.

Definition 2 (Marginal profit)**.** Given a project $i > 1$ and a profile of signals s, define the *marginal profit (MP) of dropping* i, $\mu(i, s)$, as the function $\mu(i, s) := \nu(i, \{1, \ldots, i\}, s)$.

Definition 3 (Concavity)**.** Given a profile of signals s, we say that $\pi(\{1, \ldots, i\}, s)$ is *strictly concave* in i if the MP of dropping higher-ranked projects is lower than that of lower-ranked projects: For every $i = 2, \ldots, n$, $\mu(i - 1, s) < \mu(i, s)$.

We present a modified algorithm under strict concavity. The modifications are immediate consequences of the assumption of strict concavity, so further details on the proof are omitted.

Corollary 1. *Fix a profile of signals s and relabel the projects if necessary so that* $s_1 > s_2 > \cdots > s_n$. *Assume that the function* $\pi(\{1, \ldots, i\}; s)$ *is strictly concave in i. The following algorithm constructs* $A^*(s)$.

1. *Step 1: Compute* $\mu\left(\lfloor \frac{B}{c} \rfloor, s\right)$.

   (a) *If* $\mu\left(\lfloor \frac{B}{c} \rfloor, s\right) > 0$, *set set* $A_1 := \{1, \ldots, n-1\}$ *and move to step 2.*

   (b) *If* $\mu\left(\lfloor \frac{B}{c} \rfloor, s\right) < 0$, *set* $A^*(s) := N^*$ *and stop.*

2. *Step* $k = 2, \ldots, \lfloor \frac{B}{c} \rfloor - 1$: *Compute* $\mu(\lfloor \frac{B}{c} \rfloor - k + 1, s)$.

   (a) *If* $\mu(\lfloor \frac{B}{c} \rfloor - k + 1, s) > 0$, *set* $A_k := \{1, \ldots, \lfloor \frac{B}{c} \rfloor - k\}$ *and move to step* $k + 1$.

   (b) *If* $\mu(\lfloor \frac{B}{c} \rfloor - k + 1, s) < 0$, *set* $A^*(s) := A_{k-1}$ *and stop.*

3. *Step* $\lfloor \frac{B}{c} \rfloor$: *Compute* $\pi(\{1\}, s)$.

   (a) *If* $\pi(\{1\}, s) < 0$, *set* $A^*(s) := \emptyset$ *and stop.*

   (b) *If* $\pi(\{1\}, s) > 0$, *set* $A^*(s) := \{1\}$ *and stop.*

Under strict concavity, the selection process is symmetric: A project with negative MP can be safely discarded, and the search stops as soon as the first project with positive MP is identified, provided we are within our budget. The reason is that, if dropping a project yields negative marginal profit, so will dropping all higher-ranked projects.

However, in general, the relevant incremental-profit measure is BIP, not MP. As a broader statement for managerial practice, the point is that MS are likely require block-level assessment as opposed to case-by-case, marginal assessments.

5 Independent Projects With Asymmetric Values

If projects are symmetric, we can rank them according to their signals. Comparing projects ceases to be straightforward, however, once we allow their value or signal distributions, or their costs, to be different. A more-promising project can be less appealing than a less-promising one if the latter is sufficiently cheaper.

Even if the costs are equal, asymmetric distributions render the signals no longer directly comparable. If a project is more likely to receive biased praise, a glowing report is "less good news" than a more modest report can be on another project.

In order to make comparisons across projects, the signals must first be *weighted*. If the projects are asymmetric but independent and have equal costs, we can rank them based on the conditional-expectation values $\phi_i(s_i)$. Thus, we now label the projects so that $\phi_1(s_1) > \phi_2(s_2) > \cdots > \phi_n(s_n)$, which will generally be different from the labelling based on $s_1 > s_2 > \cdots > s_n$, but then proceed as in Proposition 3 (and Corollary 1).

Example 6. There are $n = 3$ projects for consideration. Projects $i = 1, 2$ can either be successful or a dud, as in Example 1; the value for project 3, however, is as in Example 2. Thus, we have $\phi_1(s_1) = s_1$, $\phi_2(s_2) = s_2$, and $\phi_3(s_3) = \frac{s_3 - 1}{\ln(s_3)}$. Costs are $c_1 = c_2 = c_3 = 0.55$, $B = 1.1$, and $\theta = 0.5$. Let the signals be $s_1 = 0.8$, $s_2 = 0.2$, and $s_3 = 0.1$. Now, $\phi_1(s_1) = 0.8$ and $\phi_2(s_2) = 0.2$ but $\phi_3(s_3) = 0.3909 > \phi_2(s_2)$. Even though project 3 has the lowest signal, project 2 is the least-promising one once the signals are properly weighted. Figure 4 identifies the optimal portfolio as the portfolio of the top-two projects, which in this case are projects 1 and 3.

If we allow for the asymmetric projects to be correlated, but their costs remain equal, then a simpler procedure than that described in Proposition 1 leads to the optimal project portfolio. (Proof details are omitted.)

Proposition 4. *Assume that* $c_1 = \cdots = c_n = c$. *The manager's problem is equivalent to maximizing* $\pi(A, s)$ *subject to the constraint* $|A| = \lfloor \frac{B}{c} \rfloor$.

With both asymmetric distributions and asymmetric costs, Lemma 2 ceases to hold: A project that yields a high gross value and high spillover to other projects may not be profitable if its cost is too high. In this richer environment, we are back in Proposition 1.

Example 7. There are $n = 3$ projects. Projects 1 is as in Example 1; project 2 is an in Example 2; for project 3, we have that $v_3 \sim U[0, 1]$ but s_3 given v_3 is drawn from the conditional distribution with cdf $F(s_3|v_3) = (s_3/v_3)^2$ on $[0, v_3]$. Compared to the uniform distribution, the latter puts more weight on lower signals. Thus, a higher value of s_3 is "better news" about v_3 than the same value for s_2 is about v_2.

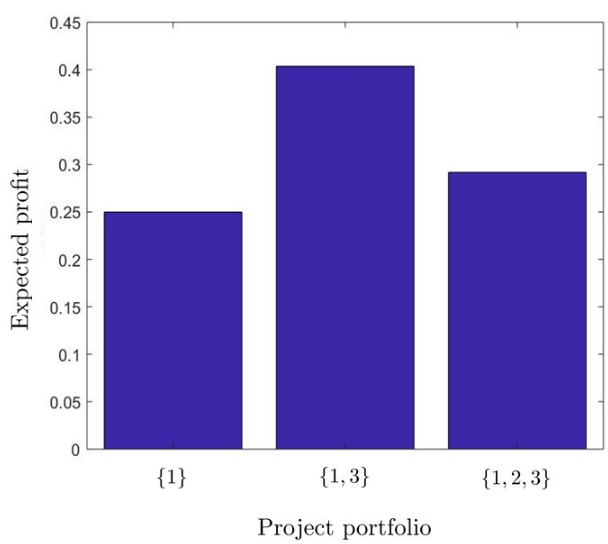

Figure 4: Profit from portfolios $\{1\}$, $\{1, 3\}$, and $\{1, 2, 3\}$ in Example 6.

Here, $\phi_1(s_1) = s_1$, $\phi_2(s_2) = \frac{s_2-1}{\ln(s_2)}$, and $\phi_3(s_3) = -\frac{s_3}{1-s_3}\ln(s_3)$. Costs are $c_1 = 0.6$, $c_2 = 0.4$, $c_3 = 0.8$, and $\theta = 0.5$. Figure 5 depicts the profit from all portfolios when signals are $s_1 = 0.9$, $s_2 = 0.3$, and $s_3 = 0.1$, and $\phi_1(s_1) = 0.9$, $\phi_2(s_2) = 0.5814$, and $\phi_3(s_3) = 0.2558$. The best portfolio is $A^*(s) = \{1,2\}$, which is feasible provided $B \geq 1$. Otherwise, we have $A^*(s) = \{1\}$ if $0.6 \leq B < 1$, and $A^*(s) = \{2\}$ if $B < 0.6$.

6 Conclusions

This paper revisits a classical problem for managers, the problem of project-portfolio selection. Although this problem has received attention in the literature, a key element of it, with tangible managerial consequences, has been overlooked: the distinction between managerial and statistical spillover.

In the absence of spillover of any kind, projects can be assessed and undertaken in complete autonomy by the corresponding unit (subject to budgetary approval). When projects' values are correlated and decisions must be made at the interim stage, on the basis of preliminary information, one project's signal becomes informative of other projects' value. This statistical spillover is consistent with decentralized project assessment and undertaking provided that the manager can ensure free information flow across divisions — so that each division can make a proper assessment of their project.

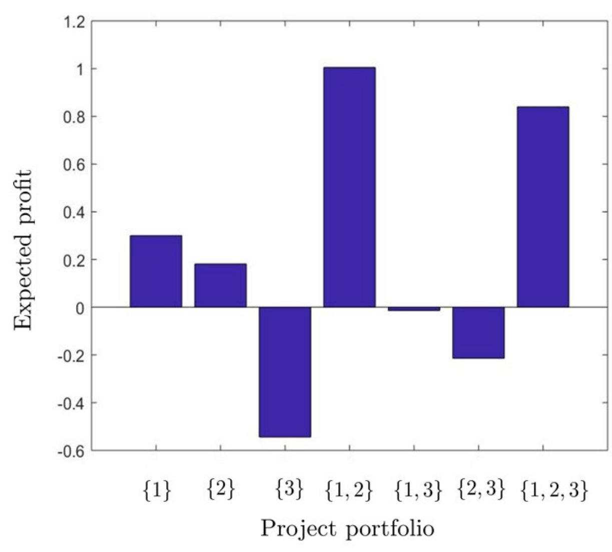

Figure 5: Expected profit from all portfolios in Example 7.

Managerial spillover, on the other hand, requires that projects be undertaken within the same unit — to ensure the exploitation of common resources and assets — and impacts how projects should be *assessed*: We must consider block-incremental profit as opposed to marginal profit.

At the aggregate level, managerial spillover provides another rationale for the firm to exist as an institution. Sharing common assets and other resources is easier under the same organizational and governance umbrella. Statistical spillover does not involve common assets or require joint authority, yet a common organization structure may facilitate the flow of information across units.

We close by emphasizing the difference, in terms of managerial practice, between investing in assets common to different businesses and the ability to use managerial "theories"or "visions" to understand correlations across the potential outcomes of different projects. The latter entails a managerial *ability to evaluate* businesses that has different, tangible managerial implications from the *ability to exploit* common assets.

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A Proofs

Proof of Lemma 1. We can identify the subsets of N with vectors in $\{0,1\}^n$, where set $A \subseteq N$ is represented as a vector a with i-th entry of 1 if $i \in A$ and of 0 otherwise. Then, we have:

$$\Pi(a,v) = \sum_{i=1}^{n} \left[a_i(v_i - c_i) + \theta \sum_{j \neq i} a_i a_j v_i v_j \right].$$

For each fixed v, the function $\Pi(a,v)$ has increasing differences. Therefore, it is supermodular. As supermodularity is preserved by taking expectation over v, the result follows. □

Proof of Proposition 1. Let A be any project portfolio such that $\sum_{i \in A} c_i \leq B$. Since $|A| \in N_B$, we have that $\pi(A,s) \leq \pi(A^*_{|A|}(s),s) \leq \pi(A^*(s),s)$, which establishes the desired result. □

Proof of Proposition 2. We can focus on sets of the form $\{i_1,\ldots,i_m\}$ for $j = 1,\ldots,m$. By construction, no such set with $m < k(s,B)$ can beat $A^0(s)$. On the other hand, sets with $m > k(s,B)$ will either not increase profit if $\phi_{i_m}(s) \leq c_{i_m}$ or be unaffordable if $\sum_{j=1}^{m} c_{i_j} > B$. Thus, no affordable portfolio can beat $A^0(s)$. □

Proof of Lemma 2. Write $\pi((A \setminus \{i\}) \cup \{j\},s) - \pi(A,s)$ as:

$$\pi((A \setminus \{i\}) \cup \{j\},s) - \pi(A;s) = \phi(s_j) - \phi(s_i) + 2\theta[\phi(s_j) - \phi(s_i)] \sum_{h \in A \setminus \{i\}} \phi(s_h)$$

By affiliation, the function $r(x) = \phi(x) + 2\theta\phi(x) \sum_{h \in A \setminus \{i\}} \phi(s_h)$ is strictly increasing. Thus, the lemma follows. □

Proof of Proposition 3. Fix signal profile s; let the algorithm terminate at portfolio $A^*(s)$, and assume that we can find a different portfolio A' such that $\pi(A', s) > \pi(A^*(s), s)$ and $|A'| \leq \lfloor \frac{B}{c} \rfloor$. By Lemma 2, we may assume that A' is of the form $A' = \{1, \ldots, j\}$ for some $j \in N$. (It cannot be empty, as otherwise we get an absurd: $0 = \pi(A', s) > \pi(A^*(s), s) \geq 0$; and if it is not of the aforementioned form, we can improve on it by swapping the lower-ranked projects in A' with the missing higher-ranked projects.) At step $n - j$, the algorithm either selects A' or identifies another feasible portfolio with an even higher payoff. Thus, if the algorithm terminates at $A^*(s)$, it must be the case that either $A^*(s) = A'$ or $\pi(A^*(s), s) > \pi(A', s)$; both of these cases lead to a contradiction. □