Trigonometry Refresher

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What it’s used for:

Trigonometry is best applied to:

• Geometry
• Astronomy
• Physics
• Advanced mathematics
• Almost any engineering field
How far to your friend’s house from your house?

Your House — b — Grocery Store

Friend’s House — a

0011 0010 1010 1101 0001 0100 1011
Triangles and Pythagorean Theorem:

Pythagorean theorem: \[ c^2 = a^2 + b^2 \]

This rule works **only** when using a **right** triangle.
(Special case of the law of cosines).
How far to your friend’s house from your house?

Your House

Grocery Store

Friend’s House

θ

b

0011 0010 1010 1101 0001 0100 1011

0011 0010 1010 1101 0001 0100 1011
Triangles and Pythagorean Theorem:

Only know the length of one side and the value of one angle.

Use trigonometry to solve for either of the unknown sides (again only for a right triangle)
Unit Circle

- "b" = length of the right triangle on the x-axis (x-value)
- "a" = height of the right triangle on the y-axis (y-value)
- "c" = distance between the origin and our point (b,a). (c = 1 for unit circle).
Trig Functions: SOHCAHTOA

\[
\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c}
\]

\[
\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c}
\]

\[
\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{a}{b}
\]

- \(a\) = opposite side to angle, \(\theta\).
- \(b\) = adjacent side to angle, \(\theta\).
- \(c\) = hypotenuse side to angle, \(\theta\).

\[\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}\]
Similar Triangles

\[ \tan(\theta) = \frac{a}{b} \]
\[ \tan(\theta) = \frac{c}{d} \]
\[ \frac{a}{b} = \frac{c}{d} \]

Same angle: ratio of sides **must** be the same!
Angles in degrees:

- There are 360 degrees in a circle.
- Some of the common angles are shown to the right.
- All positive angles start from the x-axis.
Angles in radians (standard unit):

- There are $2\pi$ radians in a circle.
- Some of the common angles are shown to the right.
- All angles start from the x-axis.
Angles in radians (standard unit):

- $2\pi = 360^\circ \Rightarrow \pi = 180^\circ$

- Calculator has radian and degree mode, so check before you calculate!
Convert these:

From degrees to radians:  From radians to degrees:

• $25^\circ$  • $\pi/7$

• $-140^\circ$  • $4$
Convert these:

From degrees to radians:

• $25^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{36}$
• $-140^\circ \times \frac{\pi}{180^\circ} = -\frac{7\pi}{9}$
• or: $-140^\circ + 360^\circ = 220^\circ$
• $220^\circ \times \frac{\pi}{180^\circ} = \frac{11\pi}{9}$

From radians to degrees:

• $\frac{\pi}{7} \times \frac{180^\circ}{\pi} \approx 25.7^\circ$
• $4 \times \frac{180^\circ}{\pi} \approx 229.2^\circ$
Try these:

\[ a = ? \]

\[ c = ? \]
Try these:

\[ \cos(30^\circ) = \frac{3}{c} \]
\[ c \cdot \cos(30^\circ) = 3 \]
\[ c = \frac{3}{\cos(30^\circ)} = 3.46 \]

\[ \tan(30^\circ) = \frac{a}{3} \]
\[ a = 3 \tan(30^\circ) = 1.73 \]
All: sine, cosine, and tangent all have positive values in this quadrant.

Star: only sine has positive values in this quadrant.

Trig: only tangent has positive values in this quadrant.

Class: only cosine has positive values in this quadrant.
Calculating Angle

Use Inverse functions to “undo” the trig functions:

- \( \sin(\theta) = 0.5 \)
  - \( \sin^{-1}(\sin(\theta)) = \sin^{-1}(0.5) \)
  - \( \theta = \sin^{-1}(0.5) = 30^\circ \)

- \( \cos(\theta) = 0.5 \)
  - \( \cos^{-1}(\cos(\theta)) = \cos^{-1}(0.5) \)
  - \( \theta = \cos^{-1}(0.5) = 60^\circ \)

- \( \tan(\theta) = 0.5 \)
  - \( \tan^{-1}(\tan(\theta)) = \tan^{-1}(0.5) \)
  - \( \theta = \tan^{-1}(0.5) = 26.57^\circ \)

\( \sin^{-1}(c) = \arcsin(c) = \text{asin}(c) \)
Trig Functions: Periodic

\( \theta = 0^\circ \Rightarrow \cos(\theta) = 1 \) and \( \sin(\theta) = 0 \): point on circle is on \( x \)-axis.

\( \theta = 90^\circ \Rightarrow \cos(\theta) = 0 \) and \( \sin(\theta) = 1 \): point on circle is on \( y \)-axis.

\( \theta = 180^\circ \Rightarrow \cos(\theta) = -1 \) and \( \sin(\theta) = 0 \): point on circle is on \( (-x) \)-axis.

\( \theta = 270^\circ \Rightarrow \cos(\theta) = 0 \) and \( \sin(\theta) = -1 \): point on circle is on \( (-y) \)-axis.

Sine and cosine increase and decrease as you go around the circle.
Trig Functions: Periodic

\[
\sin(\theta)
\]

\[
\cos(\theta)
\]
y-axis Shift

General form: 
$A + B\cos(\omega t + \phi)$

- $A$: Shifts the function up (+) or down (-) along the y-axis.

Dotted line represents the original $\cos(t)$ function.
Amplitude

General form: $A + B\cos(\omega t + \phi)$

- $B$ (Amplitude): Shrinks (small #) or stretches (large #) along the y-axis.
Frequency

General form:  
\[ A + B\cos(\omega t + \phi) \]

- \(\omega\) (frequency): Shrinks (large #) or stretches (small #) along the x-axis. (1/s = Hz)

- T (period) = \(\frac{2\pi}{\omega}\): Time it takes for the wave to complete one full cycle. (s)
Phase Shift

General form:
$A + B \cos(\omega t + \varphi)$

- $\varphi$: Shifts right (-) or left (+) along the x-axis.

If you shift cosine by $\varphi = -\pi/2$, your function becomes sine!
Common Trig Mistakes

- $\theta = \frac{x}{\sin}$

- $\sin(90^\circ) = 0.894$

Why are these wrong?
• $\sin(\theta) = \frac{b}{c}$

What's wrong with this?
• $\sin(\theta^2) = \sin^2(\theta)$

• $\sin^{-1}(\theta) = \frac{1}{\sin(\theta)}$

and

What’s wrong with this picture?
More commonly, you have a circle of radius, $r$.

\[ x^2 + y^2 = r^2 \]

\[ r^2 \cos^2(\theta) + r^2 \sin^2(\theta) = r^2 \]

\[ \cos^2(\theta) + \sin^2(\theta) = 1 \]

divide by $\cos^2(\theta)$:

\[ 1 + \frac{\sin^2(\theta)}{\cos^2(\theta)}: = \frac{1}{\cos^2(\theta)} \]

\[ 1 + \tan^2(\theta): = \sec^2(\theta) \]

$\sec(\theta) = 1/\cos(\theta)$, $\csc(\theta) = 1/\sin(\theta)$, $\cot(\theta) = 1/\tan(\theta)$
For more trig help…

Come to the QSC!

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